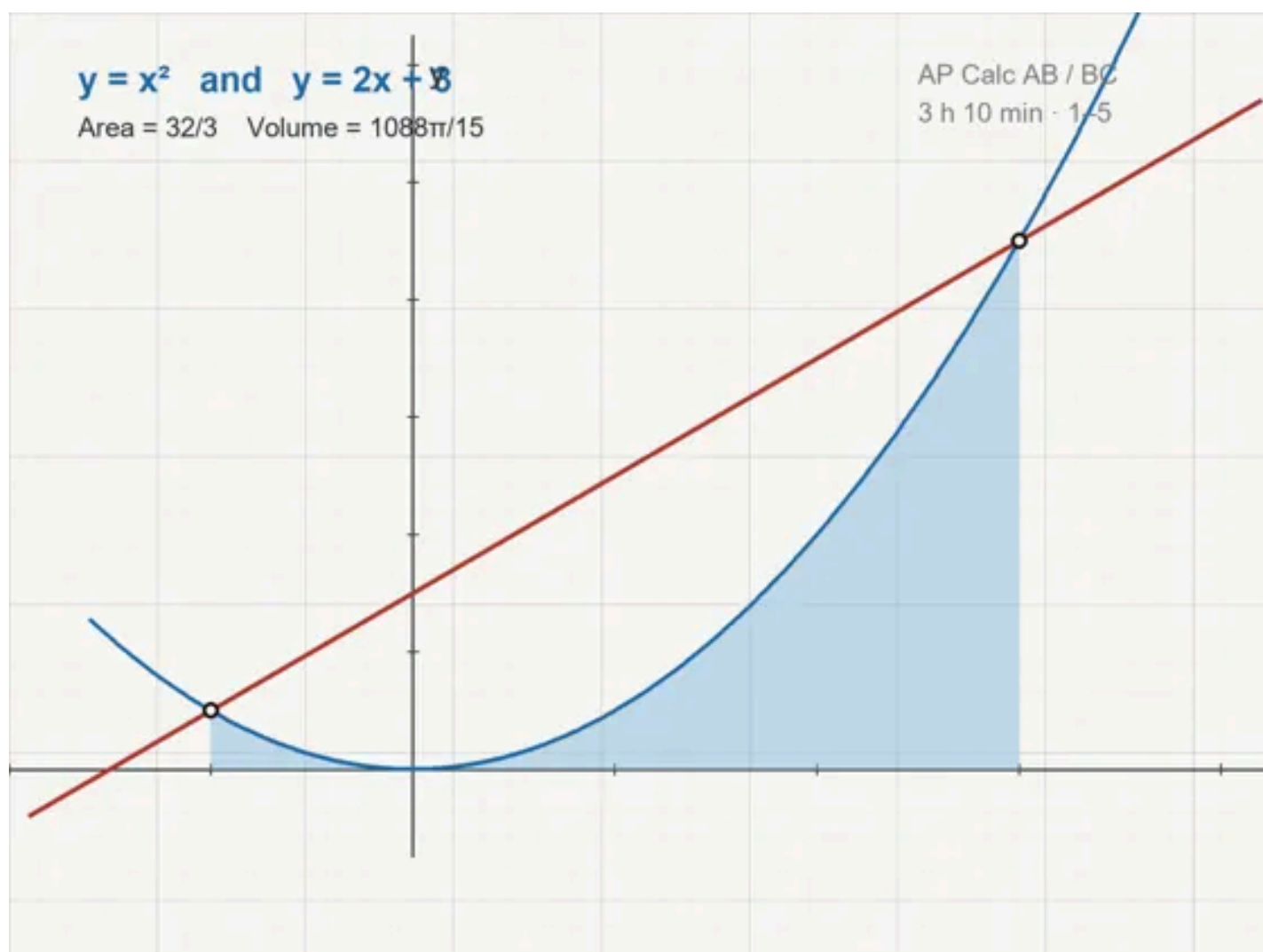


AP Calculus AB and BC Mock Paper — 42-Question Format, 3 h 10 min, with Full Worked Solutions

TutorZone (www.tutorzone.com.hk) 原創模擬試題 · 非考評局官方試題

Quick answer: AP Calculus comes in two versions, **AB** and **BC**, with an identical exam design: **3 h 10 min** in total, **Section I — 42 multiple-choice questions in 1 h 40, worth 50%** (Part A: 29 questions in 62 minutes, **no** graphing calculator, 35%; Part B: 13 questions in 38 minutes, graphing calculator **required**, 15%), and **Section II — 6 free-response questions in 1 h 30, worth 50%** (Part A: 2 questions in 30 minutes with calculator, 16.7%; Part B: 4 questions in 60 minutes **without** calculator, 33.3%). The exam is hybrid digital: questions appear in the Bluebook app and free-response answers are handwritten in paper booklets. Results are reported on a **1–5** scale, there is no coursework, and **AB and BC cannot be taken in the same year**. BC covers all of AB plus parametric, polar and vector functions, integration by parts, series and Taylor polynomials.



The graph behind free-response question 1: the curves meet at $x = -1$ and $x = 3$, giving area $32/3$ and volume $1088\pi/15$.

Exam at a glance: AP Calculus AB and BC (May 2027)

Part	Questions	Time	Calculator	Weight
Section I A (multiple choice)	29	62 min	Not permitted	35%
Section I B (multiple choice)	13	38 min	Graphing calculator required	15%
Section I total — 42 questions / 1 h 40				50%
Section II A (free response)	2	30 min	Graphing calculator required	16.7%
Section II B (free response)	4	60 min	Not permitted	33.3%
Section II total — 6 questions / 1 h 30				50%

Total exam time is **3 h 10 min**. Questions are presented in the Bluebook testing app, while free-response answers are **handwritten** in paper booklets that are collected and scored by the College Board. Calculators must be on the approved AP list (graphing calculators). Scores are reported on the **1–5** scale, and there is **no coursework**.

Four rules to remember: ① **AB and BC cannot be taken in the same year**; ② the two exams share exactly the same format, timing and weightings — only the content differs; ③ Section I Part A and Section II Part B are the **no-calculator** parts, together 68.3% of the exam, so mental and written computation decides the score; ④ scores are 1–5 and many universities grant credit or placement — check each university's own policy.

AB or BC? How to choose

Item	AP Calculus AB	AP Calculus BC
Format, timing, weightings	Identical (see the table above)	
Content	Limits and continuity, differentiation, integration, differential equations (separation of variables), areas and volumes	All of AB , plus: parametric, polar and vector functions; integration by parts, partial fractions, improper integrals; Euler's method and logistic models; arc length; sequences and series (including Taylor series)
Typical fit	One year of calculus; medicine, business, life sciences	Strong maths background; engineering, physics, mathematics, computer science
Level	Roughly one semester of university calculus	Roughly two semesters of university calculus

How to choose: if your school offers one calculus course and you are heading for engineering or physics, take BC. If time is tight, or you only need one calculus score for a business or pre-med requirement, a strong AB score is usually easier to achieve. Do not try to sit both in the same year — the College Board does not allow it.

How this paper was written — and copyright

This mock paper was written by the TutorZone tutor team against the publicly published AP Calculus AB and AP Calculus BC course and exam descriptions (exam format, timing, weightings and calculator policy). It is **not** an official AP question paper, specimen paper or past paper. AP® and its course materials remain the copyright of the College Board, and no official question or course material is reproduced here. All questions, solutions and marking points are original, and **every numerical answer has been recalculated and verified**. For self-study convenience the official 42 multiple-choice questions are represented here by **18 representative questions** (12 without calculator, 6 with), while all **6 free-response questions** are included in full, plus one BC extension question; the mark split per question is our own.

Section I mock multiple choice

Part A (12 questions, no calculator) — choose the best answer and show enough working to check yourself.

1. Find $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$. (1 mark)
2. Differentiate $x^3 \ln x$. (1 mark)
3. Evaluate $\int (3x^2 - 2x + 1) dx$. (1 mark)
4. Given $f(x) = x^3 - 6x^2 + 9x$, find the x-coordinates of all critical points of f . (1 mark)
5. Find $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$. (1 mark)
6. What is the slope of the tangent to $y = \sin 2x$ at $x = \pi/6$? (1 mark)
7. Evaluate $\int_0^2 (4 - x^2) dx$. (1 mark)
8. Find the average rate of change of $f(x) = x^2$ on $[1, 4]$. (1 mark)
9. Express $\lim_{n \rightarrow \infty} \sum_{i=1}^n (i/n)^2 (1/n)$ as a definite integral and evaluate it. (1 mark)
10. Given $f''(x) = (x - 2)(x + 1)$, find the interval on which the graph of f is concave down. (1 mark)
11. Given $f(2) = 5$ and $f'(2) = 3$, find $(f^{-1})'(5)$. (1 mark)
12. For the curve $2x^2 + 3y^2 = 30$, find dy/dx at the point $(3, 2)$. (1 mark)

Part B (6 questions, graphing calculator permitted; give numerical answers to 3 decimal places)

13. Find the average value of $f(x) = x^3$ on $[0, 2]$. (1 mark)
14. Find the area of the region enclosed by $y = x^2$ and $y = 2x$. (1 mark)
15. A particle has velocity $v(t) = 6t - 4$ (m/s) and $s(1) = 3$. Find $s(3)$. (1 mark)
16. The curve $y = \sqrt{x}$ for $x = 0$ to 4 is rotated about the x -axis. Find the volume of the solid. (1 mark)
17. A quantity grows as $y = 100e^{0.05t}$. Find the time t for y to reach 200 . (1 mark)
18. An account balance is $A(t) = 500(1.02)^t$. Find $A'(10)$, the instantaneous rate of growth in year 10 . (1 mark)

05

Section II mock free response (6 questions, plus one BC extension)

Part A (questions 1–2, graphing calculator required)

1. The region R is bounded by the curve $y = x^2$ and the line $y = 2x + 3$.
- (a) Find the points of intersection. (3 marks)
- (b) Find the area of R . (3 marks)
- (c) R is rotated about the x -axis. Find the volume of the solid, in terms of π . (3 marks)
2. A spherical balloon is inflated so that its volume increases at $100 \text{ cm}^3/\text{s}$.
- (a) Find dr/dt when the radius $r = 5 \text{ cm}$. (5 marks)
- (b) At that moment, how fast is the surface area $S = 4\pi r^2$ increasing? (4 marks)

Part B (questions 3–6, no calculator)

3. Given $f(x) = 2x^3 - 3x^2 - 12x + 4$.

- (a) Find $f'(x)$. (2 marks)
- (b) Find all critical points and classify each as a maximum or minimum. (4 marks)
- (c) Find the equation of the tangent to the curve at $x = 0$. (2 marks)
- (d) State the interval on which f is decreasing. (1 mark)

4. A curve has equation $x^2 + xy + y^2 = 7$.

- (a) Use implicit differentiation to find dy/dx . (4 marks)
- (b) Evaluate dy/dx at the point $(1, 2)$. (2 marks)
- (c) Write the equation of the tangent at that point. (2 marks)
- (d) Find the two points on the curve where the tangent is vertical. (1 mark)

5. Define $F(x) = \int_1^x (t^2 - 4) dt$.

- (a) Find $F'(x)$. (2 marks)
- (b) Locate the extremum of F on the real line and classify it. (3 marks)
- (c) Find $F(2)$. (2 marks)
- (d) How many solutions does $F(x) = 0$ have on $[1, 5]$? Justify your answer. (2 marks)

6. (a) Solve the differential equation $dy/dx = 2xy$ given that $y(0) = 3$. (6 marks)

- (b) Find $y(0.5)$. (3 marks)

BC extension (optional, 1 question)

7. (BC) (a) Write the first four terms of the Maclaurin series for $f(x) = e^{-x^2}$. (4 marks)

- (b) Use the series to estimate $\int_0^{0.5} e^{-x^2} dx$ to 4 decimal places. (3 marks)

(c) Determine whether $\sum_{n=1}^{\infty} 1/(n \cdot 3^n)$ converges, and state the test you use. (2 marks)

Section I answers (questions 1–18)

1. Cancel the factor: the expression becomes $\lim (x + 3) = 6$.
2. Product rule: $3x^2 \cdot \ln x + x^3 \cdot (1/x) = x^2(3 \ln x + 1)$.
3. $x^3 - x^2 + x + C$ (integrate term by term).
4. $f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$, so $x = 1$ or $x = 3$.
5. Use $1 - \cos x \approx x^2/2$ (or l'Hôpital twice): the limit is $1/2$.
6. $y' = 2\cos 2x$, so $y'(\pi/6) = 2\cos(\pi/3) = 1$.
7. $[4x - x^3/3]_0^2 = 8 - 8/3 = 16/3 \approx 5.333$.
8. Average rate $= (f(4) - f(1))/(4 - 1) = (16 - 1)/3 = 5$.
9. The Riemann sum is $\int_0^1 x^2 dx = 1/3$.
10. $f'' < 0 \Rightarrow (x - 2)(x + 1) < 0 \Rightarrow -1 < x < 2$.
11. $(f^{-1})'(5) = 1/f'(2) = 1/3$.
12. Implicit differentiation: $4x + 6y \cdot y' = 0 \Rightarrow y' = -2x/(3y) \Rightarrow$ at $(3, 2)$, -1 .
13. Average value $= (1/(2 - 0)) \int_0^2 x^3 dx = (1/2)(4) = 2$.
14. Intersections $x = 0$ and $x = 2$; area $= \int_0^2 (2x - x^2) dx = 4 - 8/3 = 4/3 \approx 1.333$.
15. $s(3) = 3 + \int_1^3 (6t - 4) dt = 3 + [3t^2 - 4t]_1^3 = 3 + (15 - 1) = 19$.
16. $V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi[x^2/2]_0^4 = 8\pi \approx 25.133$.
17. $100e^{0.05t} = 200 \Rightarrow t = \ln 2 / 0.05 \approx 13.863$.
18. $A'(t) = 500(1.02)^t \ln 1.02$, so $A'(10) \approx 12.070$ per year.

Section II answers and marking points (questions 1–7)

1. (a) 3 marks: $x^2 = 2x + 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0$, so $x = -1$ and $x = 3$ (points $(-1, 1)$ and $(3, 9)$).
(b) 3 marks: area = $\int_{-1}^3 (2x + 3 - x^2) dx = [x^2 + 3x - x^3/3]_{-1}^3 = 9 - (-5/3) = 32/3 \approx 10.667$. **(c) 3 marks:** $V = \pi \int_{-1}^3 [(2x + 3)^2 - (x^2)^2] dx = \pi(364/3 - 244/5) = 1088\pi/15 \approx 227.87$. Writing $V = \pi \int (R^2 - r^2) dx$ is required for the method marks.

2. (a) 5 marks: $V = (4/3)\pi r^3 \Rightarrow dV/dt = 4\pi r^2 \cdot dr/dt$ (3); $100 = 4\pi(25) \cdot dr/dt \Rightarrow dr/dt = 1/\pi \approx 0.318$ cm/s (2).
(b) 4 marks: $S = 4\pi r^2 \Rightarrow dS/dt = 8\pi r \cdot dr/dt$ (2) = $8\pi(5)(1/\pi) = 40$ cm²/s (2).

3. (a) 2 marks: $f'(x) = 6x^2 - 6x - 12$. **(b) 4 marks:** $6x^2 - 6x - 12 = 0 \Rightarrow 6(x - 2)(x + 1) = 0 \Rightarrow x = 2$ or -1 (2); $f''(x) = 12x - 6$, so at $x = -1$, $f'' = -18 < 0$ giving a maximum at **$(-1, 11)$** , and at $x = 2$, $f'' = 18 > 0$ giving a minimum at **$(2, -16)$** (2). **(c) 2 marks:** $f(0) = 4$ and $f'(0) = -12$, so the tangent is **$y = -12x + 4$** . **(d) 1 mark:** $f' < 0$ on **$-1 < x < 2$** .

4. (a) 4 marks: $2x + y + xy' + 2yy' = 0$ (3) $\Rightarrow y' = -(2x + y)/(x + 2y)$ (1). **(b) 2 marks:** at $(1, 2)$: $-(2 + 2)/(1 + 4) = -4/5$. **(c) 2 marks:** $y - 2 = -(4/5)(x - 1)$, i.e. **$y = -(4/5)x + 14/5$** . **(d) 1 mark:** a vertical tangent needs $x + 2y = 0$; substituting gives $3y^2 = 7$, so the points are **$(-2\sqrt{7/3}, \sqrt{7/3})$ and $(2\sqrt{7/3}, -\sqrt{7/3})$** (about $(-3.055, 1.528)$ and $(3.055, -1.528)$).

5. (a) 2 marks: by the Fundamental Theorem, $F'(x) = x^2 - 4$. **(b) 3 marks:** $F' = 0$ at $x = \pm 2$; **$x = -2$ is a maximum** (F' changes from $+$ to $-$) and **$x = 2$ is a minimum** (from $-$ to $+$). **(c) 2 marks:** $F(2) = [t^3/3 - 4t]_1^2 = (8/3 - 8) - (1/3 - 4) = -5/3 \approx -1.667$. **(d) 2 marks:** two solutions — $F(1) = 0$, and since F decreases to $-5/3$ and $F(5) = [t^3/3 - 4t]_1^5 = 76/3 > 0$, there is exactly one more solution in $(2, 5)$.

6. (a) 6 marks: separate the variables: $dy/y = 2x dx$ (2); $\ln|y| = x^2 + C$ (2); $y = 3e^{x^2}$ after using $y(0) = 3$ (2). **(b) 3 marks:** $y(0.5) = 3e^{0.25} \approx 3.852$.

7. (BC) (a) 4 marks: using $e^u = 1 + u + u^2/2 + u^3/6 + \dots$ with $u = -x^2$: **$1 - x^2 + x^4/2 - x^6/6$** . **(b) 3 marks:** integrate term by term: $0.5 - 0.5^3/3 + 0.5^5/10 - 0.5^7/42 \approx 0.4613$. **(c) 2 marks:** it **converges** by the ratio test, since $a_{n+1}/a_n = n/(3(n+1)) \rightarrow 1/3 < 1$; a comparison with the convergent $\sum 1/3^n$ also works because $n \cdot 3^n \geq 3^n$.

Content: AB core and BC additions

Topic	AB	BC only
Limits and continuity	Limit definitions, one-sided limits, continuity, squeeze theorem	l'Hôpital's rule for indeterminate forms; improper integrals
Differentiation	Definitions, power/product/quotient/chain rules, implicit differentiation, related rates	Parametric and vector-valued differentiation, arc length, area with polar curves
Applications of derivatives	Mean value theorem, extrema and concavity, optimisation, rates of change	Logistic models, Euler's method
Integration	Riemann sums, Fundamental Theorem, substitution, applications (area, volume, accumulation)	Integration by parts, partial fractions, improper integrals, arc length

Topic	AB	BC only
Differential equations	Separation of variables, slope fields, exponential growth and decay	Logistic differential equations, Euler's method
Sequences and series	Not examined	Convergence and divergence, ratio/comparison/integral/alternating tests, Taylor and Maclaurin series, error bounds

09

Where AP Calculus candidates lose marks

- **Ignoring the calculator rules:** only Section I Part B and Section II Part A allow the graphing calculator. Using a calculator where it is banned scores nothing, and failing to use it where it is allowed wastes minutes.
- **No setup written down:** free-response questions are scored point by point, so a volume question needs $V = \pi \int (R^2 - r^2) dx$ and a related-rates question needs $dV/dt = 4\pi r^2 \cdot dr/dt$ before any arithmetic.
- **Wrong precision:** calculator questions normally require **3 decimal places**; no-calculator questions normally require **exact** values such as $32/3$ and $1088\pi/15$. Rounding early shifts the final answer.
- **Missing units:** cm/s, cm^2/s , square units, per year — units are part of the answer, and related-rates questions regularly lose a point for omitting them.
- **Classifying extrema without justification:** state whether the first derivative changes sign or quote the second derivative, and label each point maximum or minimum explicitly.
- **Dropping the y' term:** in implicit differentiation the derivative of xy is $y + xy'$, and forgetting the y' is the single most common slip.
- **Answering BC content in an AB paper:** series, polar curves and integration by parts are BC only — AB candidates should skip them rather than lose time.

10

12-week revision plan

Weeks	Content	Practice focus
1–2	Limits, continuity, the derivative definition	Algebraic limit techniques; no-calculator limit questions
3–4	Differentiation rules, implicit differentiation	Ten hand-written derivatives a day, including product and chain
5	Applications: extrema, concavity, mean value theorem	Always write the justification, not just the answer
6	Related rates and optimisation	Write the model equation first, then differentiate with respect to time
7–8	Riemann sums, Fundamental Theorem, substitution	Convert limit-of-sum expressions into definite integrals

Weeks	Content	Practice focus
9	Area, volume, accumulation functions	Sketch to fix the limits; write $R^2 - r^2$ for washer questions
10	Differential equations and slope fields	Separation of variables; exponential growth models
11	Timed Section I: Part A (62 min) then Part B (38 min)	About 2 minutes per question in the no-calculator part
12	Timed full Section II (1 h 30, 2 + 4 questions)	About 15 minutes per question; write the scoring formula first
BC: add 2 weeks	Series convergence tests, Taylor series, parametric and polar	Twenty questions each on ratio, comparison and alternating tests

11

FAQ

Can I take AB and BC in the same year?

No. The College Board does not allow a candidate to take both AP Calculus AB and AP Calculus BC in the same year.

Is BC simply "AB plus one topic"?

BC covers all of AB and adds parametric, polar and vector functions, integration by parts and partial fractions, improper integrals, Euler's method, arc length and sequences and series including Taylor polynomials — which is why BC candidates typically need about a third more preparation time.

Which calculator may I use?

An approved graphing calculator from the College Board list, in the two calculator-permitted parts; phones, computers and unapproved models are not allowed. Check the official AP calculator policy.

Are the free-response answers typed or handwritten?

The questions are displayed in the Bluebook app, but answers are **handwritten** in paper booklets, which are collected and scored by the College Board.

How is the score calculated?

Multiple choice and free response each count 50%, combined statistically into a score of **1–5** (5 is the top). Credit and placement policies vary by university.

How does this mock paper differ from official material?

It is an original practice paper written against the published exam design. The multiple choice here is a representative subset (18 of the official 42), while all six free-response questions are included, plus one BC extension. Use it alongside the official free-response questions and scoring guidelines.

12

Official sources

Source	What to use it for
AP Calculus AB — College Board	Exam format, question counts, timing, weightings and exam dates
AP Calculus BC — College Board	BC-only content and exam design
AP Central — Calculus AB exam	Official statement of the section structure and calculator rules
AP Central — Calculus BC exam	Official BC section structure and calculator rules

See also: [A Level Mathematics 9709 Paper 1 mock paper](#), [IB Mathematics AA SL mock paper](#), [IGCSE Mathematics 0580 mock paper](#) and [exam practice questions](#). Looking for AP Calculus tuition? Start with the [subject guides](#) and then match with a tutor.