

A-Level Mathematics 9709 Paper 1 (Pure Mathematics 1) - Practice Paper

TutorZone original practice paper · 10 questions · 75 marks · 1 hour 50 minutes

Name: _____	Class: _____	Date: _____
Time allowed: 1 hour 50 minutes	Total marks: 75	Questions: 10 (25 parts)

Answer all questions. Show all necessary working; marks are given for clear method. Give non-exact numerical answers correct to 3 significant figures unless a different accuracy is stated. A calculator may not be used.

Questions

1. (6 marks) Quadratics and completing the square

- (a) Solve the equation $2x^2 - 7x + 3 = 0$. (3)
- (b) Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, and state the minimum value of $2x^2 - 12x + 7$. (3)

2. (7 marks) Binomial expansion

- (a) Find the first three terms, in ascending powers of x , of the expansion of $(1 + 2x)^8$. (3)
- (b) Find the coefficient of x^3 in the expansion. (2)
- (c) Hence estimate the value of $(1.02)^8$, giving your answer to 4 decimal places. (2)

3. (7 marks) Differentiation and stationary points

- (a) A curve has equation $y = x^3 - 6x^2 + 9x + 2$. Find dy/dx . (2)
- (b) Find the coordinates of the stationary points of the curve. (3)
- (c) Determine the nature of each stationary point. (2)

4. (8 marks) Equation of a circle

- (a) The circle with equation $x^2 + y^2 - 6x + 10y + 18 = 0$ has centre C and radius r . Find the coordinates of C and the value of r . (4)
- (b) The point P has coordinates $(10, -5)$. Find the shortest distance from P to the circle. (4)
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5. (8 marks) Trigonometric equations

- (a) Show that the equation $3\sin^2\theta - 2\cos\theta - 2 = 0$ may be written as $3\cos^2\theta + 2\cos\theta - 1 = 0$. (3)
- (b) Hence solve $3\cos^2\theta + 2\cos\theta - 1 = 0$ for $\cos\theta$. (2)
- (c) Hence solve $3\sin^2\theta - 2\cos\theta - 2 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. (3)
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6. (8 marks) Definite integration and areas

- (a) The curve $y = x^2 + 2$ crosses the lines $x = 1$ and $x = 3$. Find the area of the region bounded by the curve, the x-axis and these two lines. (4)
- (b) Find the area of the region enclosed between $y = x^2 + 2$ and $y = 3x + 2$ for $0 \leq x \leq 3$. (4)
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7. (8 marks) Geometric progressions

- (a) A geometric progression has first term 3 and common ratio $2/3$. Find the sum to infinity. (3)
- (b) Find the sum of the first 5 terms, giving your answer as an exact fraction. (3)
- (c) Find the smallest value of n for which the sum of the first n terms exceeds 8.9. (2)
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8. (8 marks) Factor theorem and factorisation

- (a) The polynomial $f(x) = 2x^3 + ax^2 + bx - 6$ has factors $(x - 1)$ and $(x + 2)$. Find the values of a and b . (5)
- (b) Hence factorise $f(x)$ completely. (3)
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9. (8 marks) Chain rule and connected rates of change

- (a) Given that $y = (3x - 1)^6$, find dy/dx and hence the value of dy/dx at $x = 1$. (4)
- (b) The radius of a circle increases at 0.5 cm per second. Find the rate of increase of the area when the radius is 6 cm, giving your answer in exact form. (4)
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10. (7 marks) Vectors

- (a) The points A and B have coordinates $(2, -1)$ and $(5, 3)$. Find the vector AB and its magnitude. (3)
- (b) Find a unit vector in the direction of AB. (2)
- (c) The vector $(k, 8)$ is parallel to AB. Find k . (2)
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Answers and mark scheme

Question 1 (6 marks)

(a) $x = 1/2$ or $x = 3$ (3)

M1 factorises correctly as $(2x - 1)(x - 3)$, or uses the formula with $a = 2$, $b = -7$, $c = 3$; A1 both roots correct.

(b) $2(x - 3)^2 - 11$; minimum value -11 (at $x = 3$) (3)

M1 takes out the factor 2 and completes the square; A1 $a = 2$, $b = -3$, $c = -11$; A1 minimum value -11 stated (allow “minimum at $x = 3$ ”).

Question 2 (7 marks)

(a) $1 + 16x + 112x^2$ (3)

M1 uses the binomial coefficients ${}^1C_1$ and ${}^1C_2$; A1 $16x$; A1 $+112x^2$.

(b) 448 (2)

M1 ${}^1C_3 \times 2^3 = 56 \times 8$; A1 448.

(c) 1.1712 (true value 1.1717) (2)

M1 substitutes $x = 0.01$ in the three terms; A1 1.1712.

Question 3 (7 marks)

(a) $3x^2 - 12x + 9$ (2)

M1 differentiates term by term; A1 $3x^2 - 12x + 9$.

(b) (1, 6) and (3, 2) (3)

M1 sets $dy/dx = 0$ and factorises $3(x - 1)(x - 3)$; A1 $x = 1, 3$; A1 both y-values correct.

(c) (1, 6) is a maximum; (3, 2) is a minimum (2)

M1 finds $d^2y/dx^2 = 6x - 12$; A1 correct classification of both points.

Question 4 (8 marks)

(a) $C(3, -5)$, $r = 4$ (4)

M1 completes the square in x and in y ; A1 centre $(3, -5)$; M1 $r^2 = 9 + 25 - 18$; A1 $r = 4$.

(b) 3 units (4)

M1 distance $CP = 7$; M1 subtracts the radius; A1 3.

Question 5 (8 marks)

(a) $3(1 - \cos^2\theta) - 2\cos\theta - 2 = 0$, then collect terms (3)

M1 uses $\sin^2\theta = 1 - \cos^2\theta$; A1 expands; A1 reaches the printed form with all signs correct.

(b) $\cos\theta = 1/3$ or $\cos\theta = -1$ (2)

M1 factorises $(3\cos\theta - 1)(\cos\theta + 1)$; A1 both values.

(c) $\theta = 70.5^\circ, 180^\circ, 289.5^\circ$ (3)

B1 $\theta = 180^\circ$; M1 $\theta = 70.5^\circ$; A1 289.5° (all to 1 d.p., no extras in range).

Question 6 (8 marks)

(a) $38/3 \approx 12.67$ square units (4)

M1 integrates to $x^3/3 + 2x$; M1 substitutes 3 and 1; A1 $15 - 7/3$; A1 $38/3$.

(b) $9/2 = 4.5$ square units (4)

M1 forms the difference $3x - x^2$; M1 integrates; A1 $27/2 - 9$; A1 $9/2$.

Question 7 (8 marks)

(a) $S_{\infty} = 9$ (3)

M1 uses $a/(1-r)$ with $r = 2/3$; A1 $3 \div (1/3) = 9$; A1 states the condition that r is between -1 and 1 .

(b) $211/27 \approx 7.815$ (3)

M1 uses $S_n = a(1-r^n)/(1-r)$; M1 substitutes $n = 5$ and $r = 2/3$; A1 $211/27$.

(c) $n = 12$ ($S_{11} \approx 8.896$, $S_{12} \approx 8.931$) (2)

M1 solves or tests the inequality; A1 $n = 12$.

Question 8 (8 marks)

(a) $a = 5$, $b = -1$ (5)

M1 $f(1) = 0$ gives $a + b = 4$; M1 $f(-2) = 0$ gives $4a - 2b = 22$; M1 solves the pair; A1 $a = 5$; A1 $b = -1$.

(b) $f(x) = (x-1)(x+2)(2x+3)$ (3)

M1 divides out one factor; A1 reaches $(x-1)(x+2)(2x+3)$; A1 all three factors correct.

Question 9 (8 marks)

(a) $dy/dx = 18(3x-1)^5$; at $x = 1$ the value is 576 (4)

M1 uses the chain rule; A1 $18(3x-1)^5$; M1 substitutes $x = 1$; A1 576 .

(b) $6\pi \approx 18.85 \text{ cm}^2 \text{ per second}$ (4)

M1 $dA/dr = 2\pi r$; M1 multiplies by dr/dt ; A1 6π ; A1 $\text{cm}^2 \text{ per second}$ with correct units.

Question 10 (7 marks)

(a) $AB = (3, 4)$; magnitude 5 (3)

M1 subtracts coordinates; A1 $(3, 4)$; A1 5 .

(b) $(3/5, 4/5)$ (2)

M1 divides AB by 5 ; A1 $(3/5, 4/5)$.

(c) $k = 6$ (2)

M1 sets up $k/3 = 8/4$; A1 $k = 6$.