

CIE A Level 數學 9709 Paper 3

CIE A Level 數學 | 時限：110 分鐘 | 總分：75 分 | 共 11 題

甲部：對數、指數方程與部分分式

全部題目均須作答。不得使用計算機。

1. Without using a calculator, simplify: (a) $\log_2 48 - \log_2 3$, (b) $\log_5 10 + \log_5 2.5$, (c) $2\log_3 6 - \log_3 4$.

答案：(a) $\log_2(48 \div 3) = \log_2 16 = 4$ [2]

(b) $\log_5(10 \times 2.5) = \log_5 25 = 2$ [2]

(c) $\log_3(6^2 \div 4) = \log_3 9 = 2$ [2]

2. Solve $4^x - 9(2^x) + 8 = 0$.

答案：Let $y = 2^x$: $y^2 - 9y + 8 = 0 \rightarrow (y - 1)(y - 8) = 0$, so $y = 1$ or $y = 8$ [3]

$2^x = 1 \rightarrow x = 0$; $2^x = 8 \rightarrow x = 3$ [2]

Check: $x = 0$ gives $1 - 9 + 8 = 0$ ✓; $x = 3$ gives $64 - 72 + 8 = 0$ ✓ [1]

3. Express $(7x + 8)/((x + 1)(x + 2))$ in partial fractions.

答案： $(7x + 8)/((x + 1)(x + 2)) \equiv A/(x + 1) + B/(x + 2)$ [1]

$7x + 8 \equiv A(x + 2) + B(x + 1)$ [1]

$x = -1$: $1 = A$; $x = -2$: $-6 = -B$ so $B = 6$ [3]

Hence $1/(x + 1) + 6/(x + 2)$; coefficients check: $A + B = 7$, $2A + B = 8$ ✓ [2]

4. $f(x) = x^3 - 4x^2 + 5x - 6$. (a) Find the remainder when $f(x)$ is divided by $(x - 2)$. (b) Given that $(x - 3)$ is a factor, factorise $f(x)$ completely and explain why the quadratic factor has no real roots.

答案：(a) Remainder = $f(2) = 8 - 16 + 10 - 6 = -4$ [2]

(b) $x^3 - 4x^2 + 5x - 6 = (x - 3)(x^2 - x + 2)$ [3]

For $x^2 - x + 2$, discriminant = $1 - 8 = -7 < 0$, so no real roots [1]

乙部：微分、積分、微分方程與數值方法

全部題目均須作答。須寫出運算步驟。

5. $f(x) = x^3 + 3x^2 - 9x + 5$. Find the coordinates of the stationary points and determine their nature.

答案： $f'(x) = 3x^2 + 6x - 9 = 3(x + 3)(x - 1) = 0 \rightarrow x = -3$ or $x = 1$ [3]

$f(-3) = -27 + 27 + 27 + 5 = 32$; $f(1) = 1 + 3 - 9 + 5 = 0$ [2]

$f''(x) = 6x + 6$; $f''(-3) = -12 < 0 \rightarrow$ maximum at $(-3, 32)$; $f''(1) = 12 > 0 \rightarrow$ minimum at $(1, 0)$ [3]

6. Evaluate $\int_1^4 (2x + 3/\sqrt{x}) dx$, giving an exact value.

答案： $\int (2x + 3x^{-1/2}) dx = x^2 + 6\sqrt{x}$ [3]

At $x = 4$: $16 + 12 = 28$; at $x = 1$: $1 + 6 = 7$ [2]

Value = $28 - 7 = 21$ [2]

7. A curve satisfies $dy/dx = 2y/x$ and passes through (1, 3). Find y in terms of x .

答案：Separate: $\int (1/y)dy = \int (2/x)dx \rightarrow \ln y = 2 \ln x + c$ [3]

$$y = Ax^2 \quad [1]$$

$$x = 1, y = 3 \rightarrow A = 3, \text{ so } y = 3x^2 \quad [2]$$

$$\text{Check: } dy/dx = 6x \text{ and } 2y/x = 6x^2/x = 6x \quad \checkmark \quad [1]$$

8. $f(x) = x^3 - 2x - 5$. (a) Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$. (b) Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2 , giving x_2 correct to 4 decimal places.

答案：(a) $f(2) = -1$ and $f(3) = 16$; sign change and f is continuous, so a root lies in (2, 3) [2]

$$f'(x) = 3x^2 - 2 \quad [1]$$

$$x_1 = 2 - (-1)/10 = 2.1 \quad [2]$$

$$f(2.1) = 0.061, f'(2.1) = 11.23 \rightarrow x_2 = 2.1 - 0.061/11.23 = 2.0946 \text{ (4 d.p.)} \quad [3]$$

丙部：三角學、向量與複數

全部題目均須作答。角度以度為單位。

9. Solve $2\sin^2\theta + 3\cos\theta = 3$ for $0^\circ \leq \theta \leq 360^\circ$.

$$\text{答案：} 2(1 - \cos^2\theta) + 3\cos\theta - 3 = 0 \rightarrow 2\cos^2\theta - 3\cos\theta + 1 = 0 \quad [2]$$

$$(2\cos\theta - 1)(\cos\theta - 1) = 0 \rightarrow \cos\theta = 1/2 \text{ or } \cos\theta = 1 \quad [2]$$

$$\cos\theta = 1/2 \rightarrow \theta = 60^\circ, 300^\circ \quad [2]$$

$$\cos\theta = 1 \rightarrow \theta = 0^\circ, 360^\circ \quad [1]$$

10. Vectors $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$. (a) Find $\mathbf{a} \cdot \mathbf{b}$. (b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

$$\text{答案：(a) } \mathbf{a} \cdot \mathbf{b} = (2)(-1) + (-1)(4) + (3)(-2) = -2 - 4 - 6 = -12 \quad [2]$$

$$|\mathbf{a}| = \sqrt{14}, |\mathbf{b}| = \sqrt{21} \quad [2]$$

$$\cos\theta = -12/\sqrt{294} = -0.6999 \rightarrow \theta = 134.4^\circ \quad [2]$$

11. $z = (4 - 2i)/(1 + i)$. Find z in the form $a + bi$, and state $|z|$ and $\arg z$ (in degrees).

$$\text{答案：Multiply by } (1 - i)/(1 - i): (4 - 2i)(1 - i)/2 = (4 - 4i - 2i + 2i^2)/2 \quad [2]$$

$$= (2 - 6i)/2 = 1 - 3i \quad [2]$$

$$|z| = \sqrt{1 + 9} = \sqrt{10} \approx 3.1623 \quad [2]$$

$$\arg z = \arctan(-3/1) \approx -71.6^\circ \quad [1]$$