

CIE A-Level 數學 9709 Paper 4

A-Level 數學 | 時限：75 分鐘 | 總分：50 分 | 共 9 題

甲部 運動學、動量與力 (25 分)

全部題目必須作答，須展示計算步驟並註明單位。

1. A particle P moves along a straight line. Its velocity is $v = 3t^2 - 12t + 9$ m/s at time t seconds. (a) Find the two values of t at which P is instantaneously at rest. (b) Find the distance travelled by P in the first 4 seconds.

答案：(a) $3t^2 - 12t + 9 = 0$ gives $t^2 - 4t + 3 = 0$, i.e. $(t - 1)(t - 3) = 0$, so $t = 1$ s and $t = 3$ s. (b) Integrating: $s = t^3 - 6t^2 + 9t$. Displacement in 4 s = 4 m. Since v changes sign at $t = 1$ s and $t = 3$ s, distance = $4 + 4 + 4 = 12$ m (displacement alone underestimates).

2. A train travelling at 30 m/s decelerates uniformly and comes to rest in 200 m. Find (a) the deceleration, (b) the time taken to stop.

答案：(a) $v^2 = u^2 + 2as$: $0 = 30^2 + 2a(200)$, so $a = -2.25$ m/s² (deceleration 2.25 m/s²). (b) $v = u + at$: $0 = 30 - 2.25t$, so $t = 13.33$ s. Check: average speed 15 m/s $\times$ 13.33 s $\approx$ 200 m.

3. Particle A of mass 3 kg moves at 5 m/s towards particle B of mass 2 kg, which is moving at 2 m/s in the opposite direction. The particles coalesce. Find (a) their common velocity, (b) the kinetic energy lost.

答案：(a) Taking A direction as positive: $3(5) + 2(-2) = 11 = 5v$, so $v = 2.2$ m/s, i.e. 2.2 m/s in the original direction of A. (b) KE before = 41.5 J; KE after = 12.1 J; loss = 29.4 J.

4. A ball is projected vertically upwards from ground level with speed 14 m/s. Taking $g = 9.8$ m/s² and ignoring air resistance, find (a) the greatest height reached, (b) the total time before the ball returns to the ground.

答案：(a) $0 = 14^2 - 2(9.8)h$, so $h = 10$ m. (b) Time to the top = $14/9.8 = 1.429$ s, so total time = 2.86 s. Check: $14(2.857) - 0.5(9.8)(2.857)^2 \approx 0$.

5. A lift of mass 800 kg is pulled upwards by a vertical cable with tension 9000 N. Find (a) its acceleration, (b) the tension needed for the lift to move upwards at constant speed.

答案：(a) $T - mg = ma$: $9000 - 800(9.8) = 800a$, so $a = 1.45$ m/s². (b) At constant speed $a = 0$, so $T = mg = 7840$ N.

乙部 力學應用 (25 分)

全部題目必須作答；取 $g = 9.8$ m/s²。

6. A block of mass 5 kg rests on rough horizontal ground. The coefficient of friction is 0.3. A horizontal force of 25 N is applied to the block. (a) Show that the block moves. (b) Find the friction force on the block. (c) Find its acceleration.

答案：(a) Greatest possible friction = $\mu R = 0.3(5)(9.8) = 14.7$ N, which is smaller than 25 N, so the block does move. (b) Friction = 14.7 N acting backwards. (c) $F - \text{friction} = ma$: $25 - 14.7 = 5a$, so $a = 2.06$ m/s².

7. A uniform rod AB has length 6 m and mass 20 kg. It rests horizontally on two supports, one at A and one at C, where $AC = 4$ m. Find the normal reaction at each support.

答案：The weight 196 N acts at the midpoint of the rod, 3 m from A. Taking moments about A: $R_C(4) = 196(3)$, so $R_C = 147$ N. Resolving vertically: $R_A + R_C = 196$, so $R_A = 49$ N.

8. A car of mass 1200 kg travels at a constant 25 m/s on a level road with the engine working at 30 kW. (a) Find the total resistance. (b) The car then climbs a straight slope with $\sin \theta = 0.05$, the power and the resistance being unchanged. Find the steady speed up the slope.

答案：(a) $P = Fv$: $30000 = F(25)$, so $F = 1200$ N. (b) Component of weight up the slope $= 1200(9.8)(0.05) = 588$ N; the total force needed is 1788 N, so $v = 30000/1788 = 16.78$ m/s.

9. Particles of mass 4 kg and 2 kg are connected by a light inextensible string which passes over a smooth fixed pulley. Both particles hang freely. Find (a) the acceleration of each particle, (b) the tension in the string.

答案：(a) $(4 - 2)g = (4 + 2)a$: $2(9.8) = 6a$, so $a = 3.27$ m/s². (b) For the lighter particle: $T - 2(9.8) = 2(3.27)$, so $T = 26.13$ N. (T is the same on both sides, as the string is light.)