

CIE A-Level 數學 9709 Paper 5

A-Level 數學 | 時限：75 分鐘 | 總分：75 分 | 共 9 題

甲部 概率、排列組合與隨機變數 (40 分)

全部題目必須作答，須展示步驟並註明所用公式。

1. The times, in minutes, taken by 40 students to travel to school are summarised by the grouped frequency table below, where each class is written as its class interval and number of students: 10-19 has 6, 20-29 has 9, 30-39 has 12, 40-49 has 8 and 50-59 has 5. (a) Using the class midpoints, calculate an estimate of the mean travelling time. (b) Calculate the standard deviation of these times. (c) The same data are to be drawn as a histogram; state the frequency density of the class 30-39.

答案：(a) Sum of $fx = 1350$ over 40 students, so the estimated mean is 33.75 minutes. (b) Sum of $fx^2 = 51640$, so the variance is $51640/40 - (33.75)^2 = 151.94$ and the standard deviation is 12.33 minutes. (c) Frequency density = frequency $\div$ class width = $12 \div 10 = 1.2$. [mean 2, standard deviation 3, frequency density 2]

2. A committee of 4 people is to be chosen from a group of 7 men and 5 women. (a) Find the number of ways in which the committee can be chosen. (b) Find the number of ways in which the committee contains exactly 2 women. (c) Find the number of ways in which the committee contains at least 3 women.

答案：(a) $C(12, 4) = 495$. (b) Choose 2 women and 2 men: $C(5, 2) \times C(7, 2) = 10 \times 21 = 210$. (c) Exactly 3 women gives $C(5, 3) \times C(7, 1) = 70$; exactly 4 women gives $C(5, 4) = 5$. Total = $70 + 5 = 75$. [3, 2, 3]

3. In a school, 45% of the students take Physics. Of the students who take Physics, 60% also take Mathematics. Of the students who do not take Physics, 25% take Mathematics. A student is chosen at random. (a) Draw a probability tree and find the probability that the student takes Mathematics. (b) Given that the student takes Mathematics, find the probability that the student also takes Physics.

答案：(a) $P(M) = (0.45)(0.60) + (0.55)(0.25) = 0.27 + 0.1375 = 0.4075$. (b) $P(P | M) = (0.45)(0.60) / 0.4075 = 0.663$ (3 s.f.). [4, 4]

4. The random variable X has probability distribution $P(X = 0) = 0.20$, $P(X = 1) = 0.35$, $P(X = 2) = 0.30$ and $P(X = 3) = 0.15$, with $P(X = x) = 0$ for all other values of x . (a) Show that this is a valid probability distribution. (b) Find $E(X)$. (c) Find $\text{Var}(X)$. (d) Find $E(3X - 2)$.

答案：(a) Each probability lies between 0 and 1 and $0.20 + 0.35 + 0.30 + 0.15 = 1.00$, so the distribution is valid. (b) $E(X) = 0(0.20) + 1(0.35) + 2(0.30) + 3(0.15) = 1.40$. (c) $E(X^2) = 2.90$, so $\text{Var}(X) = 2.90 - (1.40)^2 = 0.94$. (d) $E(3X - 2) = 3(1.40) - 2 = 2.20$. [2, 2, 2, 2]

5. The random variable X has the binomial distribution $B(20, 0.35)$. Find (a) $P(X = 6)$, (b) $P(X \leq 4)$, (c) $E(X)$ and $\text{Var}(X)$, (d) the most likely value of X .

答案：(a) $P(X = 6) = C(20, 6)(0.35)^6(0.65)^{14} = 0.171$ (3 s.f.). (b) $P(X \leq 4) = P(X = 0) + \dots + P(X = 4) = 0.118$ (3 s.f.). (c) $E(X) = 20(0.35) = 7.0$ and $\text{Var}(X) = 20(0.35)(0.65) = 4.55$. (d) The most likely value is the integer part of $(n + 1)p = 21(0.35) = 7.35$, which is 7. [2, 3, 2, 2]

乙部 二項、幾何與正態分佈 (35 分)

全部題目必須作答，正態分佈概率須寫出所用 z 值。

6. In a game a player throws a ball at a target repeatedly. The probability of hitting the target on any throw is 0.2, independently of all other throws. The random variable Y is the number of throws up to and including the first hit. (a) State the distribution of Y , giving the value of its parameter. (b) Find $P(Y = 4)$. (c) Find $P(Y > 5)$. (d) Find $E(Y)$.

答案：(a) Y has the geometric distribution with parameter 0.2. (b) $P(Y = 4) = (0.8)^3(0.2) = 0.1024$. (c) $P(Y > 5) = (0.8)^5 = 0.32768$. (d) $E(Y) = 1 \div 0.2 = 5$. [2, 2, 2, 2]

7. The masses, in grams, of apples of a certain variety are normally distributed with mean 120 and standard deviation 15. (a) Find the probability that a randomly chosen apple has a mass between 110 g and 135 g. (b) Find the mass that is exceeded by 10% of these apples. (c) Give one reason why a normal model can only be an approximation for the masses of apples.

答案：(a) $z = (110 - 120)/15 = -0.667$ and $z = (135 - 120)/15 = 1.00$, so the probability is $\Phi(1.00) - \Phi(-0.667) = 0.8413 - 0.2525 = 0.589$ (3 s.f.). (b) The 90th percentile has $z = 1.282$, so the mass is $120 + 15(1.282) = 139.2$ g. (c) The normal distribution is symmetric and unbounded, whereas masses cannot be negative and are usually slightly skew, so the model only approximates the data. [3, 3, 3]

8. Packets of tea are filled by a machine. The masses, in grams, of the packets are normally distributed with mean 250 and standard deviation 4. (a) Find the probability that a packet has a mass less than 245 g. (b) Find the probability that a packet has a mass between 248 g and 254 g. (c) The machine is adjusted so that the standard deviation changes while the mean stays at 250 g. Find the standard deviation for which 2% of packets have a mass less than 245 g.

答案：(a) $z = (245 - 250)/4 = -1.25$, so $P = \Phi(-1.25) = 0.1056$. (b) $z = -0.5$ and $z = 1.00$, so $P = 0.8413 - 0.3085 = 0.5328$. (c) $P(X \text{ less than } 245) = 0.02$ needs $z = -2.0537$, so $\sigma = (245 - 250)/(-2.0537) = 2.43$ g (3 s.f.). [3, 3, 3]

9. The heights, in cm, of 200 boys at a school are modelled by a normal distribution with mean 168 and standard deviation 7. (a) Estimate the number of boys whose height is between 160 cm and 180 cm. (b) Estimate the number of boys whose height is more than 176 cm. (c) Two boys are chosen at random. Find the probability that both are taller than 176 cm.

答案：(a) $z = (160 - 168)/7 = -1.143$ and $z = (180 - 168)/7 = 1.714$, so $P = 0.9568 - 0.1265 = 0.830$; the expected number is $200(0.830) = 166$. (b) $z = (176 - 168)/7 = 1.143$, so $P = 0.127$ and the expected number is $200(0.127) = 25$ (nearest integer). (c) Assuming the heights are independent, the probability is $(0.127)^2 = 0.016$ (3 s.f.). [3, 3, 3]