

IGCSE 附加數學 0606 Paper 1 模擬卷

IGCSE 數學 | 時限：120 分鐘 | 總分：80 分 | 共 10 題

甲部 代數、對數與圓弧量度 (36 分)

全部題目必須作答，須展示步驟；除另有指明外，答案須準確至三位有效數字。

1. (a) Simplify $\sqrt{50} + \sqrt{18} - \sqrt{8}$, giving your answer in the form $k\sqrt{2}$. (b) Evaluate $27^{(2/3)}$. (c) Show that $6 \div (\sqrt{5} - 1) = 3(\sqrt{5} + 1) \div 2$.

答案：(a) $\sqrt{50} = 5\sqrt{2}$, $\sqrt{18} = 3\sqrt{2}$ and $\sqrt{8} = 2\sqrt{2}$, so the sum is $6\sqrt{2}$. (b) $27^{(1/3)} = 3$ and $3^2 = 9$, so $27^{(2/3)} = 9$. (c) Multiply top and bottom by $(\sqrt{5} + 1)$; the denominator becomes $5 - 1 = 4$, so $6(\sqrt{5} + 1) \div 4 = 3(\sqrt{5} + 1) \div 2 \approx 4.854$. [2, 2, 2]

2. (a) Express $f(x) = 2x^2 - 12x + 7$ in the form $a(x - h)^2 + k$ and state the minimum value of f . (b) Solve $f(x) = 0$, giving your answers in surd form.

答案：(a) $f(x) = 2(x - 3)^2 - 11$, so the minimum value is -11 at $x = 3$. (b) $2(x - 3)^2 = 11$ gives $x - 3 = \pm \sqrt{22} \div 2$, so $x = 0.655$ or $x = 5.345$, i.e. $x = 3 \pm \sqrt{22} \div 2$. [3, 4]

3. $p(x) = x^3 + ax^2 + bx - 6$, where a and b are constants. $(x - 1)$ is a factor of $p(x)$, and $p(x)$ leaves a remainder of 12 when divided by $(x + 2)$. (a) Find the values of a and b . (b) Factorise $p(x)$ completely.

答案：(a) $p(1) = 0$ gives $a + b = 5$; $p(-2) = 12$ gives $2a - b = 13$. Solving: $a = 6$, $b = -1$. (b) $p(x) = (x - 1)(x^2 + 7x + 6) = (x - 1)(x + 1)(x + 6)$. [5, 3]

4. (a) Solve $\log_2 (x + 3) + \log_2 (x - 3) = 4$. (b) Solve $3^{(2x - 1)} = 5$, giving x correct to 3 significant figures.

答案：(a) The logs combine to $\log_2 (x^2 - 9) = 4$, so $x^2 - 9 = 16$, $x^2 = 25$ and $x = 5$; $x = -5$ is rejected because $x - 3$ must be positive. (b) $2x - 1 = \log_3 5$, so $x = (1 + \ln 5 \div \ln 3) \div 2 = 1.232$. [4, 3]

5. A sector OAB of a circle has radius 9 cm and angle AOB = 1.2 radians. Find (a) the length of the arc AB, (b) the area of the sector, (c) the area of the minor segment cut off by the chord AB, correct to 2 decimal places.

答案：(a) Arc = $r\theta = 9 \times 1.2 = 10.8$ cm. (b) Area = $\frac{1}{2}r^2\theta = 0.5 \times 81 \times 1.2 = 48.6$ cm². (c) Triangle OAB = $\frac{1}{2}r^2 \sin \theta = 0.5 \times 81 \times \sin 1.2 = 37.75$ cm², so the segment = $48.6 - 37.75 = 10.85$ cm² (the chord is 10.16 cm). [2, 3, 3]

乙部 三角、二項式、排列組合、向量與微積分 (44 分)

全部題目必須作答；數值答案須準確至三位有效數字，角度以弧度表示者另予指明。

6. (a) Express $3 \sin \theta + 4 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where R and α are positive constants and α is in radians, and state the maximum value of $3 \sin \theta + 4 \cos \theta$. (b) Hence solve $3 \sin \theta + 4 \cos \theta = 2$ for $0 \leq \theta \leq 2\pi$, giving your answers correct to 3 significant figures.

答案：(a) $R = \sqrt{(3^2 + 4^2)} = 5$ and $\tan \alpha = 4 \div 3$, so $\alpha = 0.927$ rad; $3 \sin \theta + 4 \cos \theta = 5 \sin(\theta + 0.927)$, whose maximum value is 5. (b) $\sin(\theta + \alpha) = 0.4$ gives $\theta + \alpha = 0.412$ or 2.730 , so $\theta = 1.80$ or $\theta = 5.77$. [4, 4]

7. (a) Find the coefficient of x^3 in the expansion of $(2 + x/3)^6$. (b) Find the coefficient of x^4 in the same expansion. (c) State the constant term.

答案：(a) ${}^6C_3 \times 2^3 \times (1/3)^3 = 20 \times 8 \div 27 = 5.926$, so the coefficient is $160/27$. (b) ${}^6C_4 \times 2^2 \times (1/3)^4 = 15 \times 4 \div 81 = 20/27 \approx 0.7407$. (c) The constant term is $2^6 = 64$. [3, 2, 2]

8. A committee of 5 is chosen from 6 boys and 5 girls. (a) How many committees contain exactly 3 girls? (b) If a committee of 5 is chosen at random, find the probability that it contains exactly 3 girls. (c) How many committees contain more girls than boys?

答案：(a) ${}^5C_3 \times {}^6C_2 = 10 \times 15 = 150$. (b) ${}^{11}C_5 = 462$, so the probability is $150 \div 462 = 25/77 \approx 0.325$. (c) ${}^{15}C_4 + ({}^5C_4 \times {}^6C_1) + {}^5C_5 = 150 + 30 + 1 = 181$. [3, 2, 3]

9. Relative to an origin O, the position vectors of A and B are $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Find (a) $\mathbf{a} \cdot \mathbf{b}$, (b) the exact values of $|\mathbf{a}|$ and $|\mathbf{b}|$, (c) the angle between $\mathbf{a}$ and $\mathbf{b}$ in degrees correct to 1 decimal place, (d) a unit vector parallel to $\mathbf{a} - \mathbf{b}$, in exact form.

答案：(a) $\mathbf{a} \cdot \mathbf{b} = 3 - 4 - 2 = -3$. (b) $|\mathbf{a}| = \sqrt{14} \approx 3.742$ and $|\mathbf{b}| = \sqrt{18} \approx 4.243$. (c) $\cos \theta = -3 \div \sqrt{252} \approx -0.1890$, so $\theta = 100.9^\circ$. (d) $\mathbf{a} - \mathbf{b} = 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ with length $\sqrt{38}$, so the unit vector is $(2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) \div \sqrt{38}$. [2, 2, 3, 3]

10. A particle moves in a straight line so that its displacement s metres from a fixed point O at time t seconds is $s = t^3 - 6t^2 + 9t$ for $t \geq 0$. (a) Find an expression for the velocity and the times at which the particle is instantaneously at rest. (b) Find the acceleration at $t = 2$ s. (c) Find the total distance travelled in the first 4 seconds.

答案：(a) $v = ds/dt = 3t^2 - 12t + 9 = 3(t - 1)(t - 3)$, so $v = 0$ at $t = 1$ s and $t = 3$ s. (b) $a = dv/dt = 6t - 12$, so at $t = 2$ s the acceleration is 0 m s^{-2} (the velocity is minimum there). (c) $s(0) = 0$, $s(1) = 4$, $s(3) = 0$ and $s(4) = 4$, so the total distance is $4 + 4 + 4 = 12 \text{ m}$ (the displacement is only 4 m). [3, 2, 6]