

IGCSE 數學 0580 Paper 4

IGCSE (Extended) 數學 | 時限：150 分鐘 | 總分：130 分 | 共 31 題

甲部 數字與代數 (40 分)

全部題目必須作答，須展示計算步驟。

1. A car costing \$24,800.00 loses 12% of its value in the first year and 9% of the remaining value in the second year. Work out its value at the end of the second year.

答案：\$24,800.00 $\times$ 0.88 = \$21,824.00; \$21,824.00 $\times$ 0.91 = \$19,859.84

2. Work out $(3.6 \times 10^{-4}) \div (8 \times 10^{-7})$, giving your answer in standard form.

答案：\$3.6 \times 10^{-4} \div 8 \times 10^{-7} = 450 = 4.5 \times 10^2\$

3. \$5,400.00 is divided between Amy and Ben in the ratio 4 : 5. Work out how much each receives.

答案：One part = \$5,400.00 $\div$ 9 = \$600.00; Amy = 4 $\times$ \$600.00 = \$2,400.00; Ben = 5 $\times$ \$600.00 = \$3,000.00

4. A town has a population of 84,000. The population increases by 2.5% each year. Work out the population after 3 years, to the nearest whole number.

答案：\$84,000 \times 1.025^3 = 90458.81\$, so about 90,459 people

5. In a sale a jacket is reduced by 20% and now costs \$144.00. Work out the original price.

答案：\$144.00 $\div$ 0.8 = \$180.00

6. A bus travels 168 km in 2 hours 20 minutes. Work out its average speed in km/h, and the time it takes to travel 240 km at that speed.

答案：\$168 \div (140 \div 60) = 72\$ km/h; \$240 \div 72 = 3.3333\$ h = 200 minutes = 3 h 20 min

7. Simplify $(2x^3)^2 \times 3x^{-4}$.

答案：\$4x^6 \times 3x^{-4} = 12x^2\$ (check $x = 2$: \$48 = 48\$)

8. The price of a tablet including 8% tax is \$486.00. Work out the price before tax.

答案：\$486.00 $\div$ 1.08 = \$450.00

9. Solve the simultaneous equations $3x + 2y = 23$ and $5x - 2y = 17$.

答案：Adding: \$8x = 40\$, $x = 5$; then \$2y = 23 - 15 = 8\$, $y = 4$. Check $25 - 8 = 17$$

10. Expand and simplify $(2x - 5)(3x + 4)$.

答案：\$6x^2 + 8x - 15x - 20 = 6x^2 - 7x - 20\$

11. A length is 4.7 cm correct to 1 decimal place. Write down the lower and upper bounds of the length.

答案：Lower bound 4.65 cm, upper bound 4.75 cm

12. An exchange rate is 1 Hong Kong dollar = 0.128 US dollars. (a) Change HK\$3,900 to US dollars. (b) Change US\$640 to HK dollars.

答案：(a) \$3,900 \times 0.128 = \\$499.20\$; (b) \$640 \div 0.128 = \\$5,000.00\$

乙部 圖形、空間與度量 (36 分)

如無特別說明，答案須附單位；除特別指明外取 $\pi = 3.142$ 。

13. A sector of a circle of radius 12 cm has angle 75° . Using $\pi = 3.142$, work out the area of the sector.

答案： $75/360 \times 3.142 \times 144 = 94.26 \text{ cm}^2$

14. A closed cylinder has base radius 7 cm and height 15 cm. Using $\pi = 3.142$, work out (a) its volume and (b) its total surface area.

答案： (a) $3.142 \times 49 \times 15 = 2309.37 \text{ cm}^3$; (b) $2 \times 3.142 \times 49 + 2 \times 3.142 \times 7 \times 15 = 967.74 \text{ cm}^2$

15. In a right-angled triangle the hypotenuse is 13 cm and one shorter side is 5 cm. Work out the third side and the angle opposite the 5 cm side, to 1 decimal place.

答案： Third side = $\sqrt{169 - 25} = 12 \text{ cm}$; $\sin \theta = 5/13$, $\theta = 22.6^\circ$

16. Two triangles are similar. The area of the smaller is 48 cm^2 and the ratio of corresponding lengths is 3 : 5. Work out the area of the larger triangle.

答案： $48 \times 25 \div 9 = 133.33 \text{ cm}^2$

17. ABCD is a cyclic quadrilateral. Angle A = 108° . Work out angle C, giving a reason for your answer.

答案： Angle C = $180^\circ - 108^\circ = 72^\circ$ (opposite angles of a cyclic quadrilateral sum to 180°)

18. A solid cone has base radius 9 cm and vertical height 12 cm. Using $\pi = 3.142$, work out (a) the slant height, (b) the volume and (c) the curved surface area.

答案： (a) $\sqrt{81 + 144} = 15 \text{ cm}$; (b) $\frac{1}{3} \times 3.142 \times 81 \times 12 = 1018.01 \text{ cm}^3$; (c) $3.142 \times 9 \times 15 = 424.17 \text{ cm}^2$

19. In triangle ABC, angle A = 40° , angle B = 65° and BC = 12 cm. Using the sine rule, work out AC, to 3 significant figures.

答案： $AC = 12 \times \sin 65^\circ \div \sin 40^\circ = 16.92 \text{ cm} \approx 16.9 \text{ cm}$

20. A map has scale 1 : 25,000. (a) Two schools are 6.4 cm apart on the map. Work out the real distance in km. (b) State the scale in the form 1 cm : n m.

答案： (a) $6.4 \times 25,000 = 160,000 \text{ cm} = 1600 \text{ m} = 1.6 \text{ km}$; (b) 1 cm : 250 m

丙部 代數與圖像 (30 分)

須展示代數步驟；圖像題須寫出座標。

21. A straight line passes through (2, -1) and (6, 7). Work out its gradient and its equation in the form $y = mx + c$.

答案： Gradient = $(7 + 1)/(6 - 2) = 2$; $-1 = 2 \times 2 + c$ so $c = -5$; $y = 2x - 5$

22. Solve $x^2 - 6x + 2 = 0$, giving your answers correct to 2 decimal places.

答案： $x = (6 \pm \sqrt{36 - 8}) \div 2 = 3 \pm \sqrt{7}$, so $x = 5.65$ or $x = 0.35$

23. Write $x^2 - 8x + 11$ in the form $(x - p)^2 + q$ and hence write down the coordinates of the minimum point of the graph.

答案 : $(x - 4)^2 - 5$; minimum point (4, -5)

24. $f(x) = 3x - 2$ and $g(x) = x^2 + 1$. Work out (a) $fg(2)$, (b) $gf(2)$ and (c) the value of x for which $g^{-1}(x) = 5$.

答案 : (a) $g(2) = 5$, $f(5) = 13$; (b) $f(2) = 4$, $g(4) = 17$; (c) $g^{-1}(x) = \sqrt{x - 1}$, $\sqrt{x - 1} = 5$ gives $x = 25$

25. Solve the inequality $4x - 7$ is less than $2x + 5$. Write down the largest positive integer that satisfies it, and how many positive integers satisfy it.

答案 : $2x$ is less than 12, so x is less than 6; largest positive integer 5; there are 5 positive integers (1 to 5)

26. y is inversely proportional to x . When $x = 3$, $y = 8$. (a) Find the constant of proportionality. (b) Work out y when $x = 9$, to 2 decimal places.

答案 : (a) $y = k/x$ so $k = 8 \times 3 = 24$; (b) $y = 24 \div 9 = 2.67$

丁部 概率與統計 (24 分)

須寫出計算過程；概率可用分數或小數表示。

27. The table shows the number of goals scored by a team in 30 matches. Goals 0, 1, 2, 3, 4 with frequencies 4, 7, 9, 6, 4. Work out (a) the mean number of goals, to 2 decimal places, and (b) the median.

答案 : (a) $\Sigma fx = 59$, $\Sigma f = 30$, mean $= 59 \div 30 = 1.97$; (b) the 15th and 16th values are both 2, so the median is 2

28. A bag contains 5 red, 3 blue and 2 green counters. Two counters are taken without replacement. Work out the probability that the first is red and the second is blue, as a fraction in its lowest terms.

答案 : $5/10 \times 3/9 = 15/90 = 1/6 (\approx 0.167)$

29. The probability that Ann is late is 0.6. If she is late, the probability that Ben is late is 0.5; if she is not late, the probability that Ben is late is 0.3. Work out the probability that (a) both are late and (b) exactly one of them is late.

答案 : (a) $0.6 \times 0.5 = 0.3$; (b) $0.6 \times 0.5 + 0.4 \times 0.3 = 0.4$

30. A scatter graph of 20 students' hours of revision against test score shows points lying close to a line going down from left to right. Describe the type and strength of the correlation, and state what it means in context.

答案 : Negative (inverse) correlation, fairly strong; in general students who revise more hours scored lower on this test — note that correlation does not prove causation

31. A cumulative frequency curve is drawn from 40 values with cumulative frequencies 4, 13, 27, 36, 40 at the upper class boundaries 10, 20, 30, 40, 50. Work out (a) the median, (b) the interquartile range, to 1 decimal place.

答案 : (a) 20th value: $20 + (20 - 13)/14 \times 10 = 25.0$; (b) $LQ = 10 + 6/9 \times 10 = 16.7$, $UQ = 30 + 3/9 \times 10 = 33.3$, $IQR = 16.7$