

Cambridge International AS & A Level — Further Mathematics (9231)

Paper 1 Further Pure Mathematics 1

Time allowed: 2 hours Total: 75 marks

8 questions

Answer **all** questions. Name: _____ Class: _____

Show all necessary working; answers without working may not receive full marks.

1. The cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ has roots α , β and γ . **[8]**

(a) Find $\Sigma\alpha^2$ and $\Sigma\alpha^2\beta^2$. (4 marks)

(b) Find the cubic equation, with integer coefficients, whose roots are $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. (4 marks)

2. The matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. **[9]**

(a) Find the two eigenvalues of A and an eigenvector for each. (6 marks)

(b) Find A^{-1} and hence solve $4x + y = 9$, $2x + 3y = 7$. (3 marks)

3. (a) Using the standard summation formulae, express $\Sigma(r = 1 \text{ to } n) r(2r + 3)$ in terms of n . (4 marks) **[8]**

(b) Using partial fractions and the method of differences, prove that $\Sigma(r = 1 \text{ to } n) \frac{1}{((2r - 1)(2r + 1))} = \frac{n}{(2n + 1)}$. (4 marks)

4. Let $z = 2 + 2\sqrt{3}i$. **[10]**

(a) Find $|z|$ and $\arg z$, giving the argument as a multiple of π . (3 marks)

(b) Find z^3 in the form $a + bi$. (3 marks)

(c) Hence solve $w^3 = -64$, giving the three roots in the form $r(\cos \theta + i \sin \theta)$. (4 marks)

5. The plane Π has equation $2x - y + 2z = 9$. The point A is $(1, -1, 3)$ and the line l has equation $\mathbf{r} = (3, 1, 2) + t(1, -2, 2)$. [10]
- (a) Show that A lies on Π and find the point where l meets Π . (4 marks)
- (b) Find the perpendicular distance from B(5, 1, 4) to Π . (3 marks)
- (c) Find the angle θ between l and Π , giving $\sin \theta$. (3 marks)
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6. Prove by induction that, for all positive integers n , $\sum_{r=1}^n r 2^r = (n-1)2^{n+1} + 2$. [10]
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7. Let $f(x) = \sqrt{1+2x}$. [10]
- (a) Find $f'(x)$, $f''(x)$ and $f'''(x)$. (4 marks)
- (b) Hence find the Maclaurin expansion of $f(x)$, up to and including the term in x^3 . (3 marks)
- (c) Use the expansion in (b) to estimate $\sqrt{1.2}$ and state the difference from the true value in scientific notation. (3 marks)
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8. The polar curve C has equation $r = a(1 + \cos \theta)$, $0 \leq \theta \leq 2\pi$, where $a > 0$. [10]
- (a) Show that the area enclosed by C is $(3/2)\pi a^2$. (6 marks)
- (b) Taking $a = 2$, find this area (exact value and to 3 significant figures) and the value of r at $\theta = \pi/3$. (4 marks)
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