HKDSE Mathematics Extended Part Module 1 Mock Paper (10 questions, 100 marks, 2 hours 15 minutes)

中六（DSE 延伸部分單元一 M1） Mathematics | 時限：135 分鐘 | 總分：100 分 | 共 10 題

Section A (Questions 1 to 5, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working in calculations, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

1. A factory production line produces items of which 12%% are defective, independently of one another. A random sample of 10 items is taken and X is the number of defective items in the sample. (a) Explain why X can be modelled by a binomial distribution and write down the distribution of X. (b) Find the probability that exactly 2 items are defective, correct to four decimal places. (c) Find the probability that at least one item is defective, correct to four decimal places. (d) Find the mean and the variance of X.

答案：(a) Each item has only two possible outcomes, defective or not defective, the probability of a defective item is constant at 0.12, the items are independent and the number of trials is fixed at 10, so X ~ B(10, 0.12). (b) P(X = 2) = C(10,2) x 0.12^2 x 0.88^8 = 45 x 0.0144 x 0.359635 = 0.2330. (c) P(X >= 1) = 1 - P(X = 0) = 1 - 0.88^10 = 1 - 0.2785 = 0.7215. (d) Mean E(X) = np = 10 x 0.12 = 1.2; variance Var(X) = np(1 - p) = 10 x 0.12 x 0.88 = 1.056. Marking notes: (a) 2 marks (two outcomes, constant probability, independence, fixed number of trials), (b) 2 marks, (c) 2 marks, (d) 2 marks.

2. A telephone support centre receives on average 3.2 calls in any 5-minute period during peak hours, and calls arrive randomly and independently. Let Y be the number of calls received in a given 5-minute period. (a) Write down the distribution of Y and its parameter, and state two assumptions needed for a Poisson model. (b) Find the probability that no call is received in a 5-minute period, correct to three decimal places. (c) Find the probability of receiving at most 2 calls in a 5-minute period, correct to three decimal places. (d) Find the probability of receiving 3 or more calls in a 5-minute period. (e) Find the mean number of calls in a 10-minute period and explain why a Poisson distribution is still suitable.

答案：(a) Y ~ Po(3.2). Assumptions: calls occur randomly at a constant average rate throughout the period, and the numbers of calls in disjoint periods are independent. (b) P(Y = 0) = e^(-3.2) = 0.041. (c) P(Y <= 2) = e^(-3.2)(1 + 3.2 + 3.2^2 / 2) = 0.380. (d) P(Y >= 3) = 1 - P(Y <= 2) = 1 - 0.380 = 0.620. (e) The mean number of calls in 10 minutes is 3.2 x 2 = 6.4. The Poisson parameter is proportional to the length of the interval, and the constant-rate and independence conditions still hold over a longer fixed interval, so Po(6.4) still describes the counts. Marking notes: (a) 2 marks, (b) 2 marks, (c) 2 marks, (d) 1 mark, (e) 2 marks.

3. The mock examination marks X of a class of Form 6 students are approximately normally distributed with mean 62 and standard deviation 8. Given that Phi(1) = 0.8413, Phi(-1) = 0.1587 and Phi inverse of 0.90 = 1.2816. (a) Find the proportion of students scoring above 70. (b) Find the proportion of students scoring between 54 and 70. (c) The top 10% of students receive a scholarship. Find the minimum mark needed, correct to one decimal place. (d) If there are 240 candidates, estimate the number scoring below 54. (e) Explain why the answer to (d) is only an estimate and not an exact number of students.

答案：(a) z = (70 - 62) / 8 = 1, so P(X > 70) = 1 - 0.8413 = 0.1587, about 15.87%. (b) z for 54 is (54 - 62) / 8 = -1 and z for 70 is 1, so P(54 < X < 70) = 0.8413 - 0.1587 = 0.6826, about 68.26%. (c) If the mark is s, then P(X > s) = 0.10 gives z = 1.2816, so s = 62 + 1.2816 x 8 = 72.3 marks. (d) P(X < 54) = 0.1587, so the estimated number is 240 x 0.1587 = 38.1, about 38 students. (e) The marks are only approximately normal, so the model differs from the true distribution, and the 240 candidates form a finite sample. Multiplying a probability by the number of candidates gives an expected value only, so the result is an estimate. Marking notes: (a) 2 marks, (b) 2 marks, (c) 3 marks, (d) 2 marks, (e) 1 mark.

4. Let f(x) = x^3 - 6x^2 + 9x + 2. (a) Find f prime of x and f double prime of x. (b) Solve f prime of x = 0 and find the coordinates of all stationary points of y = f(x). (c) Use the second derivative test to classify each stationary point and state the corresponding extreme value. (d) Find the coordinates of the point of inflexion and explain whether it is an extreme point. (e) Find the equation of the tangent to the curve at x = 0 and describe the behaviour of f on [0, 3].

答案：(a) f prime = 3x^2 - 12x + 9 and f double prime = 6x - 12. (b) 3x^2 - 12x + 9 = 0, that is 3(x - 1)(x - 3) = 0, so x = 1 or x = 3. Then f(1) = 1 - 6 + 9 + 2 = 6 and f(3) = 27 - 54 + 27 + 2 = 2, so the stationary points are (1, 6) and (3, 2). (c) f double prime at 1 is 6 - 12 = -6 < 0, so (1, 6) is a maximum point with maximum value 6. f double prime at 3 is 18 - 12 = 6 > 0, so (3, 2) is a minimum point with minimum value 2. (d) f double prime = 0 gives x = 2 and f(2) = 8 - 24 + 18 + 2 = 4, so the point of inflexion is (2, 4). Since f prime is still negative near x = 2 (it equals -3 there), only the concavity changes and the point is not an extreme point. (e) f(0) = 2 and f prime at 0 is 9, so the tangent is y = 9x + 2. On [0, 1] f prime is positive so f is increasing, and on [1, 3] f prime is negative so f is decreasing; the largest value on the interval is 6. Marking notes: (a) 2 marks, (b) 3 marks, (c) 2 marks, (d) 2 marks, (e) 2 marks.

5. Consider the curve C: y = x^2 - 2x and the line L: y = x. (a) Find the points of intersection of C and L. (b) Find the area of the region enclosed by C and L. (c) Find the area of the region enclosed by C and the x-axis between x = 0 and x = 2. (d) In (c) the definite integral of x^2 - 2x from 0 to 2 is negative. Explain what this means for the area. (e) If the region in (b) is rotated about the x-axis, write down, but do not evaluate, the definite integral for the volume of the solid formed.

答案：(a) Setting x^2 - 2x = x gives x^2 - 3x = 0, that is x(x - 3) = 0, so x = 0 or x = 3 and the points of intersection are (0, 0) and (3, 3). (b) For 0 < x < 3 the line lies above the curve, so the area is the integral from 0 to 3 of (x - (x^2 - 2x)) = (3x - x^2), which gives 27/2 - 9 = 4.5 square units. (c) The integral of x^2 - 2x from 0 to 2 equals 8/3 - 4 = -4/3, so the area is 4/3 square units, about 1.33. (d) For 0 < x < 2 the curve lies below the x-axis, so the definite integral gives a signed area; the negative sign shows the region is below the axis, and the area is the absolute value, 4/3. (e) Using washers, the volume is pi times the integral from 0 to 3 of (x^2 - (x^2 - 2x)^2) dx. Marking notes: (a) 3 marks, (b) 3 marks, (c) 3 marks, (d) 2 marks, (e) 1 mark.

Section B (Questions 6 to 10, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working in calculations, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

6. Consider the definite integral of 1/(1 + x^2) from 0 to 1, whose exact value is pi/4 = 0.785398... (a) Taking f(x) = 1/(1 + x^2) and subinterval width h = 0.25, complete the table of f(x) at x = 0, 0.25, 0.5, 0.75 and 1, correct to six decimal places. (b) Use the trapezoidal rule with n = 4 to estimate the integral, correct to six decimal places. (c) Find the absolute error of the estimate. (d) Explain why increasing n reduces the error and state whether the rule overestimates or underestimates here. (e) Write the general trapezoidal rule formula in terms of h, f0, f1, ..., fn.

答案： (a) f(0) = 1.000000; f(0.25) = 1 / 1.0625 = 0.941176; f(0.5) = 1 / 1.25 = 0.800000; f(0.75) = 1 / 1.5625 = 0.640000; f(1) = 1 / 2 = 0.500000. (b) T = h/2 x [f0 + f4 + 2(f1 + f2 + f3)] = 0.125 x [1 + 0.5 + 2(0.941176 + 0.800000 + 0.640000)] = 0.125 x 6.262353 = 0.782794. (c) The error is 0.785398 - 0.782794 = 0.002604. (d) As n increases each subinterval shortens, so replacing the curve by a straight segment introduces less deviation; the error is of order h squared, so it falls. Here the rule underestimates, since 0.782794 is less than the true value 0.785398. (e) T = h/2 x [f0 + 2f1 + 2f2 + ... + 2fn-1 + fn], where h = (b - a)/n. Marking notes: (a) 3 marks, (b) 3 marks, (c) 1 mark, (d) 2 marks, (e) 1 mark.

7. A factory produces a component on two machines. Machine A makes 60 per cent of the output and 2 per cent of its items are defective; machine B makes the other 40 per cent and 5 per cent of its items are defective. One item is chosen at random from the total output. (a) Use a tree diagram or probability formula to find the probability that the item is defective. (b) Given that the item is defective, find the probability that it came from A, correct to three decimal places. (c) Given that the item is defective, find the probability that it came from B, and check that the answers to (b) and (c) add to 1. (d) Given that the item is not defective, find the probability that it came from B, correct to three decimals. (e) Decide whether the events that the item came from A and that the item is defective are independent, justifying your answer with numbers.

答案： (a) P(defective) = 0.6 x 0.02 + 0.4 x 0.05 = 0.012 + 0.020 = 0.032. (b) P(A | defective) = (0.6 x 0.02) / 0.032 = 0.012 / 0.032 = 0.375. (c) P(B | defective) = (0.4 x 0.05) / 0.032 = 0.020 / 0.032 = 0.625, and 0.375 + 0.625 = 1. (d) P(not defective) = 0.6 x 0.98 + 0.4 x 0.95 = 0.588 + 0.380 = 0.968, so P(B | not defective) = 0.380 / 0.968 = 0.393. (e) P(A) x P(defective) = 0.6 x 0.032 = 0.0192 whereas P(A and defective) = 0.012, so the product rule for independence fails and the events are not independent: items from A are less likely to be defective. Marking notes: (a) 2 marks, (b) 2 marks, (c) 2 marks, (d) 2 marks, (e) 2 marks.

8. To estimate the population mean mark mu of a school, a random sample of 50 students gives a sample mean of 72.4 and a sample standard deviation of 10.5. Assume the population standard deviation can be estimated by s, that the sample mean is approximately normally distributed, and that z at 0.025 is 1.96. (a) State the point estimate of mu. (b) Find the standard error of the sample mean, correct to three decimal places. (c) Find the 95 per cent confidence interval for mu, correct to three decimal places, and interpret it. (d) Find the smallest sample size needed for the 95 per cent confidence interval to have a margin of error of at most 2 marks. (e) Explain why the interval in (c) cannot be read as a 95 per cent probability that mu lies inside it.

答案：(a) The point estimate of mu is the sample mean, 72.4 marks. (b) Standard error = s / root n = 10.5 / root 50 = 1.485. (c) Margin of error = 1.96 x 1.485 = 2.910, so the interval is (72.4 - 2.910, 72.4 + 2.910) = (69.490, 75.310). It means that if sampling were repeated many times in the same way, about 95 per cent of the intervals calculated would contain the true mu. (d) Requiring 1.96 x 10.5 / root n <= 2 gives root n >= 10.29, so n >= 105.88 and at least 106 students are needed. (e) In the frequentist view mu is a fixed but unknown constant, so the interval (69.490, 75.310) is a fixed numerical interval; the 95 per cent describes the long-run success rate of the method, not a probability for mu itself. Marking notes: (a) 1 mark, (b) 2 marks, (c) 3 marks, (d) 2 marks, (e) 2 marks.

9. A study records five pairs of values of x (hours of revision per day) and y (test score): (1, 2), (2, 4), (3, 5), (4, 4) and (5, 5). (a) Find the means of x and y. (b) Find Sxx, Syy and Sxy. (c) Find b and a in the regression line y = a + bx, correct to one decimal place. (d) Find the correlation coefficient r, correct to three decimal places, and describe the strength and direction of the relationship. (e) Use the line to predict y when x = 6, and explain whether a prediction at x = 15 is appropriate.

答案：(a) The mean of x is 3 and the mean of y is 4. (b) Sxx = 4 + 1 + 0 + 1 + 4 = 10; Syy = 4 + 0 + 1 + 0 + 1 = 6; Sxy = 4 + 0 + 0 + 0 + 2 = 6. (c) b = 6 / 10 = 0.6 and a = 4 - 0.6 x 3 = 2.2, so the line is y = 2.2 + 0.6 x. (d) r = 6 / root of 60 = 0.775, which is positive and close to one, so x and y have a fairly strong positive correlation. (e) At x = 6 the prediction is 5.8. A value of 15 lies far outside the observed range from 1 to 5, so the prediction is an extrapolation and the fitted relationship need not hold there. Marking notes: (a) 2 marks, (b) 3 marks, (c) 2 marks, (d) 2 marks, (e) 1 mark.

10. A discrete random variable X has the distribution P(X = 0) = 0.1, P(X = 1) = 0.3, P(X = 2) = 0.4 and P(X = 3) = 0.2. (a) Show that this is a valid probability distribution. (b) Find E(X). (c) Find E(X squared) and Var(X). (d) Find E(3X + 2) and Var(3X + 2). (e) Write down the cumulative distribution function F(x) at x = 0, 1, 2 and 3, and find P(1 <= X <= 2).

答案：(a) Every probability lies between 0 and 1 and 0.1 + 0.3 + 0.4 + 0.2 = 1, so the distribution is valid. (b) E(X) = 0 x 0.1 + 1 x 0.3 + 2 x 0.4 + 3 x 0.2 = 1.7. (c) E(X squared) = 0 x 0.1 + 1 x 0.3 + 4 x 0.4 + 9 x 0.2 = 3.7, so Var(X) = 3.7 - 2.89 = 0.81. (d) E(3X + 2) = 3E(X) + 2 = 7.1 and Var(3X + 2) = 9 x Var(X) = 7.29. (e) F(0) = 0.1, F(1) = 0.4, F(2) = 0.8 and F(3) = 1, and P(1 <= X <= 2) = 0.3 + 0.4 = 0.7. Marking notes: (a) 2 marks, (b) 2 marks, (c) 2 marks, (d) 2 marks, (e) 2 marks.

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