

HKDSE Mathematics Extended Part Module 1 Mock Paper (10 questions, 100 marks, 2 hours 15 minutes)

中六 (DSE 延伸部分單元一 M1) Mathematics | 時限 : 135 分鐘 | 總分 : 100 分 | 共 10 題

Section A (Questions 1 to 5, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working in calculations, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

1. A factory production line produces items of which 12% are defective, independently of one another. A random sample of 10 items is taken and X is the number of defective items in the sample. (a) Explain why X can be modelled by a binomial distribution and write down the distribution of X . (b) Find the probability that exactly 2 items are defective, correct to four decimal places. (c) Find the probability that at least one item is defective, correct to four decimal places. (d) Find the mean and the variance of X .
2. A telephone support centre receives on average 3.2 calls in any 5-minute period during peak hours, and calls arrive randomly and independently. Let Y be the number of calls received in a given 5-minute period. (a) Write down the distribution of Y and its parameter, and state two assumptions needed for a Poisson model. (b) Find the probability that no call is received in a 5-minute period, correct to three decimal places. (c) Find the probability of receiving at most 2 calls in a 5-minute period, correct to three decimal places. (d) Find the probability of receiving 3 or more calls in a 5-minute period. (e) Find the mean number of calls in a 10-minute period and explain why a Poisson distribution is still suitable.
3. The mock examination marks X of a class of Form 6 students are approximately normally distributed with mean 62 and standard deviation 8. Given that $\Phi(1) = 0.8413$, $\Phi(-1) = 0.1587$ and $\Phi^{-1}(0.90) = 1.2816$. (a) Find the proportion of students scoring above 70. (b) Find the proportion of students scoring between 54 and 70. (c) The top 10% of students receive a scholarship. Find the minimum mark needed, correct to one decimal place. (d) If there are 240 candidates, estimate the number scoring below 54. (e) Explain why the answer to (d) is only an estimate and not an exact number of students.
4. Let $f(x) = x^3 - 6x^2 + 9x + 2$. (a) Find $f'(x)$ and $f''(x)$. (b) Solve $f'(x) = 0$ and find the coordinates of all stationary points of $y = f(x)$. (c) Use the second derivative test to classify each stationary point and state the corresponding extreme value. (d) Find the coordinates of the point of inflexion and explain whether it is an extreme point. (e) Find the equation of the tangent to the curve at $x = 0$ and describe the behaviour of f on $[0, 3]$.

5. Consider the curve $C: y = x^2 - 2x$ and the line $L: y = x$. (a) Find the points of intersection of C and L . (b) Find the area of the region enclosed by C and L . (c) Find the area of the region enclosed by C and the x -axis between $x = 0$ and $x = 2$. (d) In (c) the definite integral of $x^2 - 2x$ from 0 to 2 is negative. Explain what this means for the area. (e) If the region in (b) is rotated about the x -axis, write down, but do not evaluate, the definite integral for the volume of the solid formed.

Section B (Questions 6 to 10, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working in calculations, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

6. Consider the definite integral of $1/(1 + x^2)$ from 0 to 1, whose exact value is $\pi/4 = 0.785398\dots$ (a) Taking $f(x) = 1/(1 + x^2)$ and subinterval width $h = 0.25$, complete the table of $f(x)$ at $x = 0, 0.25, 0.5, 0.75$ and 1, correct to six decimal places. (b) Use the trapezoidal rule with $n = 4$ to estimate the integral, correct to six decimal places. (c) Find the absolute error of the estimate. (d) Explain why increasing n reduces the error and state whether the rule overestimates or underestimates here. (e) Write the general trapezoidal rule formula in terms of $h, f_0, f_1, \dots, f_n$.

7. A factory produces a component on two machines. Machine A makes 60 per cent of the output and 2 per cent of its items are defective; machine B makes the other 40 per cent and 5 per cent of its items are defective. One item is chosen at random from the total output. (a) Use a tree diagram or probability formula to find the probability that the item is defective. (b) Given that the item is defective, find the probability that it came from A, correct to three decimal places. (c) Given that the item is defective, find the probability that it came from B, and check that the answers to (b) and (c) add to 1. (d) Given that the item is not defective, find the probability that it came from B, correct to three decimals. (e) Decide whether the events that the item came from A and that the item is defective are independent, justifying your answer with numbers.

8. To estimate the population mean mark μ of a school, a random sample of 50 students gives a sample mean of 72.4 and a sample standard deviation of 10.5. Assume the population standard deviation can be estimated by s , that the sample mean is approximately normally distributed, and that z at 0.025 is 1.96. (a) State the point estimate of μ . (b) Find the standard error of the sample mean, correct to three decimal places. (c) Find the 95 per cent confidence interval for μ , correct to three decimal places, and interpret it. (d) Find the smallest sample size needed for the 95 per cent confidence interval to have a margin of error of at most 2 marks. (e) Explain why the interval in (c) cannot be read as a 95 per cent probability that μ lies inside it.

9. A study records five pairs of values of x (hours of revision per day) and y (test score): (1, 2), (2, 4), (3, 5), (4, 4) and (5, 5). (a) Find the means of x and y . (b) Find S_{xx} , S_{yy} and S_{xy} . (c) Find b and a in the regression line $y = a + bx$, correct to one decimal place. (d) Find the correlation coefficient r , correct to three decimal places, and describe the strength and direction of the relationship. (e) Use the line to predict y when $x = 6$, and explain whether a prediction at $x = 15$ is appropriate.

10. A discrete random variable X has the distribution $P(X = 0) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.4$ and $P(X = 3) = 0.2$. (a) Show that this is a valid probability distribution. (b) Find $E(X)$. (c) Find $E(X^2)$ and $\text{Var}(X)$. (d) Find $E(3X + 2)$ and $\text{Var}(3X + 2)$. (e) Write down the cumulative distribution function $F(x)$ at $x = 0, 1, 2$ and 3 , and find $P(1 \leq X \leq 2)$.