

IB Maths AA SL Paper 2 Mock Paper (10 questions, 80 marks, calculator)

IBDP (Year 12-13) Mathematics | 時限：90 分鐘 | 總分：80 分 | 共 10 題

Section A (Questions 1 to 5, 40 marks)

Answer ALL questions. A calculator (including a graphical calculator) is allowed. Unless told otherwise, give numerical answers to 3 significant figures and angles to 1 decimal place.

1. The arithmetic sequence $\{u_n\}$ has $u_1 = 7$ and common difference $d = 5$; the geometric sequence $\{v_n\}$ has $v_1 = 200$ and common ratio $r = 0.8$. (a) Find u_{20} . (b) Find the sum of the first 20 terms, S_{20} . (c) Find v_5 . (d) Find the sum to infinity S_{inf} of the geometric sequence.

答案：(a) $u_{20} = 7 + 19 \times 5 = 102$. (b) $S_{20} = 20 / 2 \times (2 \times 7 + 19 \times 5) = 10 \times 109 = 1090$. (c) $v_5 = 200 \times 0.8^4 = 200 \times 0.4096 = 81.92$. (d) Since $|r| = 0.8$ is less than 1, $S_{\text{inf}} = 200 / (1 - 0.8) = 1000$.

2. (a) Solve $3^{2x-1} = 27^{x-2}$. (b) Solve $\log_2 x + \log_2(x-2) = 3$. (c) Find the value of $\log_5(1/125)$. (d) Simplify $(\ln 8 + \ln 2) / \ln 4$. (e) Solve $2^x = 50$, giving your answer to 3 significant figures.

答案：(a) Since $27 = 3^3$, $2x - 1 = 3x - 6$, so $x = 5$. (b) The domain needs $x > 2$; $x(x-2) = 8$ gives $x^2 - 2x - 8 = 0$, $(x-4)(x+2) = 0$, so $x = 4$ ($x = -2$ rejected), hence $x = 4$. (c) $1/125 = 5^{-3}$, so the value is -3 . (d) $\ln 8 + \ln 2 = \ln 16$ and $\ln 16 / \ln 4 = \log_4 16 = 2$. (e) $x = \log_2 50 = \ln 50 / \ln 2 = 5.64$ (3 s.f.).

3. Let $f(x) = 2x^2 - 8x + 5$. (a) Find the coordinates of the vertex. (b) Find the discriminant and state the number of real roots of $f(x) = 0$. (c) Solve $f(x) = 0$, giving answers to 3 significant figures. (d) Write down the equation of the axis of symmetry and the minimum value.

答案：(a) $x = 8 / 4 = 2$ and $f(2) = 8 - 16 + 5 = -3$, so the vertex is $(2, -3)$. (b) Discriminant $= 64 - 40 = 24$, which is greater than 0, so there are two distinct real roots. (c) $x = (8 \pm \sqrt{24}) / 4 = 2 \pm 1.2247$, so $x = 3.22$ or $x = 0.775$ (3 s.f.). (d) Axis of symmetry $x = 2$; minimum value -3 .

4. In triangle ABC, $a = 8$ cm, $b = 11$ cm and angle $C = 35$ degrees. (a) Find the length of c (3 s.f.). (b) Find the area of the triangle (3 s.f.). (c) Find angle A and angle B (1 d.p.).

答案：(a) By the cosine rule, $c^2 = 64 + 121 - 176 \cos 35 = 40.829$, so $c = 6.39$ cm (3 s.f.). (b) Area $= 0.5 \times 8 \times 11 \times \sin 35 = 25.2$ cm² (3 s.f.). (c) By the sine rule, $\sin A = 8 \sin 35 / 6.390 = 0.7181$, so $A = 45.9$ degrees and $B = 180 - 35 - 45.87 = 99.1$ degrees.

5. A bag contains 6 red balls and 4 blue balls. (a) Two balls are drawn without replacement; find the probability that both are red. (b) Hence find the probability that at least one ball is blue. (c) If draws are made with replacement, find the probability that exactly 2 of 3 draws are red. (d) For 200 draws with replacement, find the expected number of red balls.

答案：(a) $6/10 \times 5/9 = 1/3 = 0.333$. (b) $1 - 1/3 = 2/3 = 0.667$. (c) $3 \times 0.6^2 \times 0.4 = 0.432$. (d) $200 \times 0.6 = 120$.

Section B (Questions 6 to 10, 40 marks)

Answer ALL questions. A calculator (including a graphical calculator) is allowed. Unless told otherwise, give numerical answers to 3 significant figures and angles to 1 decimal place.

6. The curve has equation $y = x^3 - 6x^2 + 9x$. (a) Find dy/dx and the x-coordinates of the stationary points. (b) Use the second derivative to classify each stationary point and find its y value. (c) Evaluate the integral from 0 to 2 of $(3x^2 - 4x + 1) dx$.

答案 : (a) $dy/dx = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$, so stationary points at $x = 1$ and $x = 3$. (b) $y'' = 6x - 12$; at $x = 1$, $y'' = -6$ (maximum, $y = 4$); at $x = 3$, $y'' = 6$ (minimum, $y = 0$). (c) An antiderivative is $x^3 - 2x^2 + x$, giving $2 - 0 = 2$.

7. The random variable X is normally distributed with mean 50 and standard deviation 8. (a) Find $P(X > 62)$. (b) Find $P(42 < X < 58)$. (c) Find the 90th percentile of X (2 d.p.).

答案 : (a) $Z = (62 - 50) / 8 = 1.5$, so $P(Z > 1.5) = 0.0668$. (b) Z lies between -1 and 1, so the probability is 0.6827. (c) $z = 1.2816$, so $X = 50 + 8 \times 1.2816 = 60.25$.

8. The vectors are $a = (3, -2)$ and $b = (-1, 4)$. (a) Find $a + b$ and its magnitude. (b) Find the scalar product $a \cdot b$. (c) Find the angle between a and b (1 d.p.). (d) Find the unit vector in the direction of a (3 s.f.).

答案 : (a) $a + b = (2, 2)$ with magnitude $\sqrt{8} = 2.83$. (b) $a \cdot b = 3 \times (-1) + (-2) \times 4 = -11$. (c) $|a| = 3.606$ and $|b| = 4.123$, so $\cos(\theta) = -11 / 14.866 = -0.7399$ and $\theta = 137.7$ degrees. (d) The unit vector is $(3, -2) / 3.606 = (0.832, -0.555)$.

9. The discrete random variable X has $P(X = 0) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.4$ and $P(X = 3) = 0.2$. (a) Find $E(X)$. (b) Find $E(X^2)$. (c) Find $\text{Var}(X)$. (d) Find the standard deviation of X.

答案 : (a) $E(X) = 0 \times 0.1 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.2 = 1.7$. (b) $E(X^2) = 0 + 0.3 + 1.6 + 1.8 = 3.7$. (c) $\text{Var}(X) = 3.7 - 2.89 = 0.81$. (d) Standard deviation = $\sqrt{0.81} = 0.9$.

10. The population of a town grows at 4% per year and is now 1 200. (a) Find the population after 10 years (nearest integer). (b) Find the number of years needed for the population first to exceed 2 000. (c) 5 000 dollars is invested at 3.6% per year compounded quarterly; find the amount after 4 years (2 d.p.).

答案 : (a) $1200 \times 1.04^{10} = 1776$ (nearest integer). (b) $1200 \times 1.04^t > 2000$ gives $t > \ln(5/3) / \ln 1.04 = 13.02$, so 14 years are needed. (c) $5000 \times (1 + 0.036/4)^{16} = 5770.70$ dollars.