

HKDSE Mathematics Extended Part Module 2 (Algebra and Calculus) Mock Paper (10 questions, 100 marks, 2 hours 30 minutes)

中六 (HKDSE 選修單元) Mathematics | 時限 : 150 分鐘 | 總分 : 100 分 | 共 10 題

Section A (Questions 1 to 6, 50 marks)

Answer ALL questions. Show your working, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

- Let $S(n)$ be the statement "for every positive integer n , the sum of r^2 from $r = 1$ to n equals $(n - 1) \cdot 2^{(n+1)} + 2$ ".
(a) Prove $S(n)$ by mathematical induction for all positive integers n .
(b) Hence find the sum of r^2 from $r = 1$ to 5.
- In the expansion of $(2 - x/2)^8$,
(a) find the constant term;
(b) find the coefficient of x^3 ;
(c) find the coefficient of x^4 .
- (a) Find the limit of $(x^2 - 5x + 6)/(x - 2)$ as x tends to 2.
(b) Let $h(x) = x^2 \ln x$ for x greater than 0. Find $h'(1)$.
(c) Find the equation of the tangent to $y = h(x)$ at $x = 1$.
- (a) Use the substitution $u = x^2 + 3$ to find the integral of $x\sqrt{x^2 + 3}$ with respect to x .
(b) Hence evaluate the definite integral of $x\sqrt{x^2 + 3}$ from $x = 1$ to $x = 3$, giving the answer to three significant figures.
- (a) Solve $2\sin^2(\theta) - \sin(\theta) - 1 = 0$ for $0 \leq \theta \leq 360$ degrees.
(b) Hence solve $2\sin^2(2x) - \sin(2x) - 1 = 0$ for $0 \leq x \leq 180$ degrees.
- Let A be the matrix with rows $(2, 1)$ and $(3, 4)$.
(a) Find $\det A$ and A^{-1} .
(b) Using the inverse matrix, solve $2x + y = 5$ and $3x + 4y = 6$.

Section B (Questions 7 to 10, 50 marks)

Answer ALL questions. Show your working, give numerical answers to three significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

- Consider the system $x + y + z = 6$, $2x - y + z = 3$ and $x + 2y - kz = 3$, where k is a real number.
(a) Express the determinant of the coefficient matrix in terms of k .
(b) Find the value of k for which the system does not have a unique solution.
(c) When $k = 1$, solve the system by Cramer's rule.

8. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$.
(a) Find $|\mathbf{a}|$, $|\mathbf{b}|$ and the scalar product $\mathbf{a} \cdot \mathbf{b}$.
(b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$, giving the answer to three significant figures.
(c) Find the vector product $\mathbf{a} \times \mathbf{b}$ and $|\mathbf{a} \times \mathbf{b}|$.
(d) Find the scalar component of $\mathbf{b}$ along $\mathbf{a}$.
9. The curve C has equation $y = x^3 - 6x^2 + 9x + 1$.
(a) Find the stationary points of C and determine their nature.
(b) Find the point of inflexion of C .
(c) Find the area of the region bounded by C , the lines $x = 0$, $x = 3$ and the x -axis.
10. (a) Use integration by parts to find the integral of $x^2 e^{2x}$ with respect to x .
(b) Hence evaluate the definite integral of $x^2 e^{2x}$ from $x = 0$ to $x = 1$, giving the answer to three significant figures.
(c) Explain why the substitution $u = e^{2x}$ cannot be used directly for part (a).