

IGCSE Mathematics 0580 Paper 4 (Extended) Mock Paper 2 (12 questions, 100 marks, 1 hour 45 minutes)

中四至中五 (IGCSE Extended) Mathematics | 時限：105 分鐘 | 總分：100 分 | 共 12 題

Section A (Questions 1 to 6, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working, give numerical answers to 3 significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

1. A jacket is marked at \$560. In a sale every item is reduced by 15%. (a) Find the sale price of the jacket. (b) Ming and his sister buy the jacket together and share the cost in the ratio 3 : 4. How much does each of them pay?

答案：(a) Sale price = $560 \times (1 - 0.15) = \476.00 . (4 marks) (b) One part = $476.00 / 7 = \$68.00$; Ming pays 3 parts = \$204.00 and his sister pays 4 parts = \$272.00. (4 marks)

2. (a) Write 0.00046 in standard form $a \times 10^n$, where 1 is at most a and a is less than 10. (b) Work out $(3.2 \times 10^5) \times (4.5 \times 10^{-3})$, giving your answer in standard form. (c) $x = 6.3$ correct to 1 decimal place and $y = 2.8$ correct to 1 decimal place. Find the greatest possible value of x / y , correct to 3 significant figures.

答案：(a) $0.00046 = 4.6 \times 10^{-4}$. (2 marks) (b) $3.2 \times 4.5 = 14.4$ and $10^5 \times 10^{-3} = 10^2$, so the answer is $1.44 \times 10^3 (= 1440)$. (3 marks) (c) Upper bound of $x = 6.35$, lower bound of $y = 2.75$, so the greatest value is $6.35 / 2.75 = 2.309$. (3 marks)

3. (a) Expand $(3x - 2)(x + 4)$. (b) Factorise $2x^2 + 5x - 3$. (c) Hence solve the equation $2x^2 + 5x - 3 = 0$.

答案：(a) $(3x - 2)(x + 4) = 3x^2 + 12x - 2x - 8 = 3x^2 + 10x - 8$. (2 marks) (b) $2x^2 + 5x - 3 = (2x - 1)(x + 3)$. (3 marks) (c) $2x - 1 = 0$ or $x + 3 = 0$, so $x = 0.5$ or $x = -3$. (3 marks)

4. The first term of a sequence is 7 and each later term is 4 more than the term before it. (a) Write down the first four terms. (b) Find the 20th term. (c) Find the sum of the first 20 terms.

答案：(a) 7, 11, 15, 19. (2 marks) (b) n th term = $7 + (n - 1) \times 4$; 20th term = $7 + 19 \times 4 = 83$. (3 marks) (c) Sum = $20 / 2 \times (7 + 83) = 900$. (3 marks)

5. (a) Mr Chan deposits \$4000 in a bank at 3.5% per year, with interest compounded once a year. Find the amount after 4 years, correct to the nearest cent. (b) How much interest is earned in the 4 years? (c) The variable p is inversely proportional to the square of q . When $q = 3$, $p = 8$. Find p when $q = 4$.

答案：(a) Amount = $4000 \times (1 + 0.035)^4 = \4590.09 . (4 marks) (b) Interest = $4590.09 - 4000 = \$590.09$. (2 marks) (c) $p \propto q^{-2}$ is constant, so $k = 8 \times 9 = 72$; when $q = 4$, $p = 72 / 16 = 4.5$. (3 marks)

6. (a) Find the interior angle, the exterior angle and the angle sum of a regular 12-sided polygon. (b) In a circle AB is a diameter and C is a point on the circle. Given that angle ABC = 35 degrees, find angle BAC. (c) ABCD is a cyclic quadrilateral with angle ABC = 108 degrees. Find angle ADC and state the theorem used.

答案 : (a) Angle sum = $(12 - 2) \times 180 = 1800$ degrees; each interior angle = $1800 / 12 = 150$ degrees; each exterior angle = $360 / 12 = 30$ degrees. (3 marks) (b) The angle in a semicircle is 90 degrees, so angle ACB = 90 degrees and angle BAC = $180 - 90 - 35 = 55$ degrees. (3 marks) (c) Opposite angles of a cyclic quadrilateral add to 180 degrees, so angle ADC = $180 - 108 = 72$ degrees. (3 marks)

Section B (Questions 7 to 12, 50 marks)

Answer ALL questions. A calculator may be used, but it must not have graphing or symbolic algebra functions. Show your working, give numerical answers to 3 significant figures unless told otherwise, and state units in every final answer. All data are examples and this is not a past paper of any examination board.

7. In triangle ABC, AB = 7 cm, AC = 9 cm and angle BAC = 62 degrees. (a) Find the length of BC, correct to 3 significant figures. (b) Find the area of triangle ABC, correct to 3 significant figures.

答案 : (a) Cosine rule: $BC^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \times \cos 62 \text{ degrees} = 49 + 81 - 126 \times 0.4695 = 70.847$, so BC = 8.417 cm. (5 marks) (b) Area = $\frac{1}{2} \times 7 \times 9 \times \sin 62 \text{ degrees} = \frac{1}{2} \times 63 \times 0.8829 = 27.81 \text{ cm}^2$. (4 marks)

8. A solid cylinder has radius 6 cm and height 10 cm. (a) Find the volume of the cylinder, correct to 3 significant figures. (b) Find the total surface area of the cylinder, correct to 3 significant figures. (c) The cylinder is melted and recast into a sphere. Can the sphere fit completely inside the original cylinder? Give a reason.

答案 : (a) Volume = $\pi \times 6^2 \times 10 = 1130.97 \text{ cm}^3$, that is $1.13 \times 10^3 \text{ cm}^3$ to 3 s.f. (3 marks) (b) Total surface area = $2\pi \times 6^2 + 2\pi \times 6(10) = 226.19 + 376.99 = 603.19 \text{ cm}^2$, that is 603 cm^2 to 3 s.f. (4 marks) (c) $\frac{4}{3}\pi r^3 = 1130.97$ gives $r = 6.46 \text{ cm}$, which is greater than 6 cm, so the sphere cannot fit completely inside the cylinder. (2 marks)

9. The numbers of books read in a month by 11 students are given in order: 12, 15, 15, 18, 20, 22, 22, 25, 28, 30, 34 (a) Find the mean, the median and the mode. (b) Find the lower quartile, the upper quartile and the interquartile range. (c) Every student reads 2 more books. Write down the new mean and the new interquartile range.

答案 : (a) Total = 241, mean = $241 / 11 = 21.91$ books; median (6th value) = 22 books; modes = 15 and 22 books. (4 marks) (b) Lower quartile = 15, upper quartile = 28, interquartile range = $28 - 15 = 13$. (3 marks) (c) New mean = $21.91 + 2 = 23.91$ books; the interquartile range is unchanged at 13. (2 marks)

10. A bag contains 5 red balls and 3 blue balls. Ming takes one ball at random, notes its colour and does not replace it, then takes a second ball. (a) Draw a tree diagram and write the probability on each branch. (b) Find the probability that both balls are red. (c) Find the probability that the two balls are of different colours. (d) Find the probability that at least one ball is red.

答案 : (a) First draw: red $\frac{5}{8}$, blue $\frac{3}{8}$. Second draw: 4 red and 3 blue remain out of 7, so the branches are $\frac{4}{7}$ and $\frac{3}{7}$, or $\frac{5}{7}$ and $\frac{2}{7}$. (2 marks) (b) $P(\text{red, red}) = \frac{5}{8} \times \frac{4}{7} = 0.357$. (2 marks) (c) $P(\text{different colours}) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7} = 0.536$. (3 marks) (d) $P(\text{at least one red}) = 1 - P(\text{blue, blue}) = 1 - \frac{3}{8} \times \frac{2}{7} = 0.893$. (2 marks)

11. The points A(1, 2) and B(7, 10) are given. (a) Find the coordinates of the midpoint of AB. (b) Find the gradient of AB. (c) Find the equation of the line L through A that is perpendicular to AB, in the form $y = mx + c$. (d) L meets the line $y = 2x - 3$ at P. Find the coordinates of P, correct to 3 significant figures.

答案 : (a) Midpoint = $((1+7)/2, (2+10)/2) = (4, 6)$. (2 marks) (b) Gradient = $(10 - 2) / (7 - 1) = 1.3333$. (2 marks) (c) Perpendicular gradient = $-1 / 1.3333 = -0.75$; $y - 2 = -0.75(x - 1)$, so $y = -0.75x + 2.75$. (2 marks) (d) Solving $-0.75x + 2.75 = 2x - 3$ gives $x = 2.0909$ and $y = 1.1818$, so $P = (2.09, 1.18)$. (2 marks)

12. Let $f(x) = 2x - 3$ and $g(x) = x^2 - 4$. (a) Find $f^{-1}(x)$. (b) Find $fg(3)$ and $gf(3)$. (c) A car accelerates from rest to 12 m/s in 10 seconds, travels at this speed for 15 seconds, then decelerates uniformly to rest in 5 seconds. Find the total distance travelled.

答案 : (a) Let $y = 2x - 3$, then $x = (y + 3) / 2$, so $f^{-1}(x) = (x + 3) / 2$. (2 marks) (b) $g(3) = 9 - 4 = 5$ and $f(5) = 7$, so $fg(3) = 7$; $f(3) = 3$ and $g(3) = 5$, so $gf(3) = 5$. (2 marks) (c) Distance = $\frac{1}{2} \times 10 \times 12 + 15 \times 12 + \frac{1}{2} \times 5 \times 12 = 60 + 180 + 30 = 270.0$ m. (2 marks)