S3 Mathematics Practice Paper: Polynomials and Identities, Quadratic Equations, Graphs of Functions, Triangles and Quadrilaterals, Properties of Circles, Statistics

Level: S3 | Subject: Mathematics | 60 minutes | 100 marks | 24 questions | answer key

Section A: Polynomials and Identities (18 marks)

Show all working. Simplify by collecting like terms, and factorise completely.

#	Question	Answer	Marks
1	Expand (2x - 3)(x + 5) and evaluate the expression when x = 2.	Multiplying term by term: 2x^2 + 10x - 3x - 15 = 2x^2 + 7x - 15. Substituting x = 2: 2(4) + 7(2) - 15 = 7.	5
2	Factorise (a) x^2 - 7x + 12, (b) 2x^2 - 5x - 3.	(a) The two numbers with sum 7 and product 12 are 3 and 4, so x^2 - 7x + 12 = (x - 3)(x - 4). (b) 2x^2 - 5x - 3 = (2x + 1)(x - 3); checking gives 2x^2 - 6x + x - 3 = 2x^2 - 5x - 3.	5
3	Given that (x + a)^2 = x^2 + 10x + b is an identity, find a and b.	Expanding the left side gives x^2 + 2ax + a^2. Comparing coefficients: 2a = 10, so a = 5; b = a^2 = 25.	4
4	Use long division to divide 2x^3 - 5x^2 - 4x + 3 by (2x - 1), and hence solve 2x^3 - 5x^2 - 4x + 3 = 0.	Long division gives quotient x^2 - 2x - 3 with zero remainder. Since x^2 - 2x - 3 = (x - 3)(x + 1), we get (2x - 1)(x - 3)(x + 1) = 0, so x = 1/2, 3 or -1.	4

Section B: Quadratic Equations (17 marks)

Any valid method is accepted; give approximate answers correct to 2 decimal places.

#	Question	Answer	Marks
5	Solve x^2 - 6x + 5 = 0.	x^2 - 6x + 5 = (x - 1)(x - 5) = 0, so x = 1 or x = 5. (Check: 1 + 5 = 6 and 1 x 5 = 5.)	5
6	Solve 2x^2 + 3x - 4 = 0 using the quadratic formula, correct to 2 decimal places.	The discriminant is 41. x = (-3 +/- sqrt(41)) / 4, so x = 0.85 or x = -2.35.	5
7	If the equation kx^2 - 8x + 2 = 0 (k not equal to 0) has equal roots, find k.	Equal roots means the discriminant is zero: 64 - 8k = 0, so k = 8.	4
8	Find two consecutive positive integers whose product is 156.	Let the smaller integer be n, then n(n + 1) = 156, so n^2 + n - 156 = 0. Factorising gives (n - 12)(n + 13) = 0; taking the positive root n = 12, so the numbers are 12 and 13.	3

Section C: Graphs of Functions (17 marks)

Show the completing-the-square steps; for intersections give both x and y coordinates.

#	Question	Answer	Marks
9	Given y = x^2 - 4x + 3, (a) find the vertex and axis of symmetry by completing the square, (b) find the intercepts with both axes.	(a) y = (x - 2)^2 - 1, so the vertex is (2, -1) and the axis is x = 2. (b) The y-intercept is 3; setting y = 0 gives (x - 1)(x - 3) = 0, so the x-intercepts are 1 and 3.	5

10	Given y = -2(x - 1)^2 + 8, find (a) the maximum value and the corresponding x, (b) the x-intercepts.	(a) Since (x - 1)^2 >= 0 and the coefficient is negative, the maximum value is 8, occurring at x = 1. (b) Setting y = 0: (x - 1)^2 = 4, so x - 1 = +/-2 and x = 3 or -1.	4
11	Find the points of intersection of the line y = 3x - 2 and the curve y = x^2.	From x^2 = 3x - 2 we get x^2 - 3x + 2 = 0, i.e. (x - 1)(x - 2) = 0. When x = 1, y = 1; when x = 2, y = 4, so the points are (1, 1) and (2, 4).	4
12	Describe the transformation that maps y = x^2 onto y = (x + 2)^2 - 5, and state the minimum point of the latter graph.	The graph is translated 2 units to the left and 5 units down. The minimum point moves from (0, 0) to (-2, -5), so the minimum value is -5.	4

Section D: Triangles and Quadrilaterals (16 marks)

State the theorem or formula used; give areas in cm^2.

#	Question	Answer	Marks
13	A triangle has sides 8 cm, 15 cm and 17 cm. (a) Show that it is right-angled; (b) find its area and perimeter.	(a) 8^2 + 15^2 = 64 + 225 = 289 = 17^2, so by the converse of Pythagoras it is right-angled. (b) The legs are 8 cm and 15 cm, so area = 0.5 x 8 x 15 = 60 cm^2 and perimeter = 40 cm.	5
14	An isosceles triangle has base 12 cm and equal sides 10 cm. Find its height and area.	The height bisects the base at right angles, so height = sqrt(10^2 - 6^2) = sqrt(100 - 36) = 8 cm. Area = 0.5 x 12 x 8 = 48 cm^2.	4
15	A rhombus has diagonals 6 cm and 8 cm. Find (a) its area, (b) its side length, (c) its perimeter.	(a) Area = 0.5 x 6 x 8 = 24 cm^2. (b) The diagonals bisect each other at right angles, so side = sqrt(3^2 + 4^2) = 5 cm. (c) Perimeter = 4 x 5 = 20 cm.	4
16	Find the sum of the interior angles of a regular 12-sided polygon and the size of each interior angle.	Sum of interior angles = (12 - 2) x 180 = 1800 degrees, so each interior angle = 1800 / 12 = 150 degrees.	3

Section E: Properties of Circles (16 marks)

Quote the circle property used; lengths in cm and angles in degrees.

#	Question	Answer	Marks
17	A circle of radius 10 cm has a chord of length 16 cm. Find the distance from the centre to the chord.	The perpendicular from the centre bisects the chord, giving a half-chord of 8 cm. Distance = sqrt(10^2 - 8^2) = sqrt(100 - 64) = 6 cm.	5
18	The angle at the centre is 130 degrees and C lies on the major arc. Find (a) the angle at the circumference subtended by AB, (b) the angle in the opposite segment.	(a) The angle at the circumference is half the angle at the centre: 130 / 2 = 65 degrees. (b) Opposite angles of a cyclic quadrilateral are supplementary: 180 - 65 = 115 degrees.	4
19	P is an external point with OP = 13 cm, the radius is 5 cm and PT is a tangent. Find PT.	The tangent is perpendicular to the radius, so angle OTP = 90 degrees. PT = sqrt(13^2 - 5^2) = sqrt(169 - 25) = 12 cm.	4
20	In a cyclic quadrilateral ABCD, angle A = 3x and angle C = 2x + 30 degrees. Find x and the sizes of these two angles.	Opposite angles of a cyclic quadrilateral are supplementary: 3x + (2x + 30) = 180, so 5x = 150 and x = 30. Angle A = 90 degrees and angle C = 90 degrees.	3

Section F: Statistics (16 marks)

Give non-integer means and standard deviations correct to 2 decimal places.

#	Question	Answer	Marks
21	The data are 12, 14, 14, 15, 18, 19, 21 and 27. Find the mean, median, mode and range.	Mean = 140 / 8 = 17.5. With 8 values the median is the average of the 4th and 5th, i.e. (15 + 18) / 2 = 16.5. The mode is 14 and the range is 27 - 12 = 15.	5
22	Find the mean and standard deviation of the data 2, 4, 6, 8 and 10.	Mean = 30 / 5 = 6. Sum of squared deviations = 16 + 4 + 0 + 4 + 16 = 40, so variance = 40 / 5 = 8 and standard deviation = sqrt(8) = 2.83.	4
23	A bag contains 5 red, 3 blue and 2 green balls. One ball is drawn at random. Find (a) the probability that it is red, (b) the probability that it is not blue.	Total number of balls = 10. (a) P(red) = 5/10 = 0.5. (b) P(not blue) = 1 - 3/10 = 0.7.	4
24	The mean of 5 numbers is 12. A number 18 is added. Find the new mean.	Original total = 5 x 12 = 60. New mean = (60 + 18) / 6 = 13.0.	3