S6 Mathematics Extended Part Module 2 Practice Paper

S6 | Mathematics (Module 2) | Time: 75 minutes | Total: 100 marks | 24 questions

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Part A: Mathematical Induction and Binomial Theorem (12 marks)

Show all working; give coefficients in simplest form.

1. Find the coefficient of x^3 in the expansion of (1 + 2x)^5.

2. Find the constant term in the expansion of (x - 3/x)^6.

3. Use mathematical induction to prove that 1 + 3 + 5 + … + (2n - 1) = n^2 for all positive integers n.

4. Find the coefficient of x^2 in the expansion of (2 + x)^7.

Part B: Trigonometric Identities and Equations (16 marks)

Unless otherwise stated, give answers to 3 significant figures or in surd form.

5. Given that A and B are acute angles with sin A = 3/5 and cos B = 5/13, find the exact values of sin(A + B) and cos(A + B).

6. Solve 2sin^2 x - sin x - 1 = 0 for 0° ≤ x ≤ 360°.

7. Find the maximum value of 3sin x + 4cos x and the value of tan x at that maximum.

8. Given cos A = 0.6, find the value of cos 2A.

Part C: Limits and Applications of Differentiation (20 marks)

Show the steps of differentiation or limit evaluation.

9. Evaluate lim(x→0) sin 3x / (5x).

10. Let y = x^2 e^(3x). Find dy/dx, and evaluate it at x = 1 (to 3 significant figures).

11. Find the maximum and minimum points of the curve y = x^3 - 3x^2 + 2.

12. Find the equation of the tangent to the curve y = x ln x at x = e.

Part D: Integration, Area and Volume (20 marks)

For definite integrals, show the antiderivative step.

13. Evaluate ∫(from 0 to 1)(3x^2 + 2x) dx.

14. Evaluate ∫(from 0 to 1) x e^x dx.

15. Find the area of the region bounded by the curves y = x^2 and y = 2x - x^2.

16. The curve y = x^2 from x = 0 to x = 1 is rotated through one complete turn about the x-axis. Find the volume of the solid generated.

Part E: Vectors (16 marks)

Let a = (2, -1, 3) and b = (1, 4, -2).

17. Find a · b.

18. Find the angle between a and b (to 3 significant figures).

19. Find a × b and its magnitude (magnitude to 3 significant figures).

20. Find a + 2b and the scalar product of this vector with b.

Part F: Matrices and Linear Transformations (16 marks)

Let A = [[2, 1], [3, 4]] and B = [[1, 2], [0, 3]].

21. Find det A and the inverse of A.

22. Find AB and hence det(AB).

23. Use the inverse matrix to solve the system 2x + y = 8, 3x + 4y = 27.

24. A linear transformation T is represented by the matrix M = [[0, -1], [1, 0]]. Describe the geometric effect of T and find its area scale factor.
