S6 Mathematics (Compulsory Part) Practice Paper: remainder and factor theorem, fractional equations and absolute value inequalities, linear programming, equations of circles and tangents, loci, arithmetic and geometric sequences, logarithmic growth, permutations and conditional probability, binomial distribution, normal distribution and regression (100 marks)

Level: S6 | Subject: Mathematics (Compulsory Part) | 75 minutes | 100 marks | 24 questions | answer key

Section A - Remainder and Factor Theorem

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
1	Let P(x) = 2x^3 - 3x^2 + ax + 5, and P(2) = 7. (a) Find a. (b) Hence find P(-1).	(a) P(2) = 16 - 12 + 2a + 5 = 7, so 2a + 9 = 7 and a = -1. (b) P(-1) = -2 - 3 + 1 + 5 = 1.	4
2	Given P(x) = x^3 - 4x^2 + x + 6. (a) Show that x - 3 is a factor of P(x). (b) Hence factorise P(x) completely.	(a) P(3) = 27 - 36 + 3 + 6 = 0, so x - 3 is a factor. (b) By long division P(x) = (x - 3)(x^2 - x - 2), and x^2 - x - 2 = (x - 2)(x + 1), so P(x) = (x - 3)(x - 2)(x + 1).	4
3	Divide 2x^3 + x^2 - 5x + 2 by x + 2, giving the quotient and the remainder.	By the remainder theorem the remainder is 2(-8) + 4 + 10 + 2 = 0. By long division the quotient is 2x^2 - 3x + 1 and the remainder is 0. Check: (x + 2)(2x^2 - 3x + 1) = 2x^3 + x^2 - 5x + 2.	4
4	Let k be a constant such that x + 1 is a factor of 2x^3 + kx^2 - 3x + 2. Find k and hence solve 2x^3 + kx^2 - 3x + 2 = 0.	(a) By the factor theorem, 2(-1) + k + 3 + 2 = 0, so k + 3 = 0 and k = -3. (b) Dividing 2x^3 - 3x^2 - 3x + 2 by x + 1 gives 2x^2 - 5x + 2 = (2x - 1)(x - 2), so x = -1, x = 1/2 or x = 2.	4

Section B - Equations, Inequalities and Linear Programming

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
5	Subject to x + 2y <= 8, 2x + y <= 10, x >= 0, y >= 0, find the maximum value of P = 3x + 2y and the corresponding (x, y).	The vertices of the feasible region are (0, 0), (5, 0), (4, 2) and (0, 4). The values of P are 0, 15, 16 and 8. Hence the maximum is 16 at (x, y) = (4, 2).	4
6	Solve 4 / (x + 1) = x - 2.	Multiply both sides by (x + 1), x not equal -1: 4 = (x - 2)(x + 1) = x^2 - x - 2, so x^2 - x - 6 = 0 and (x - 3)(x + 2) = 0, giving x = 3 or x = -2. Neither equals -1, so both are solutions.	4
7	Solve the inequality |2x - 5| < 7 and list all integer solutions.	From -7 < 2x - 5 < 7 we get -2 < 2x < 12, so -1 < x < 6. The integer solutions are 0, 1, 2, 3, 4 and 5, six in total.	4
8	Solve the simultaneous equations y = x + 1 and x^2 + y^2 = 25.	Substituting gives x^2 + (x + 1)^2 = 25, so 2x^2 + 2x - 24 = 0 and x^2 + x - 12 = 0, i.e. (x - 3)(x + 4) = 0. Hence (x, y) = (3, 4) or (-4, -3).	4

Section C - Equations of Circles, Tangents and Loci

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
9	Find the centre and radius of the circle x^2 + y^2 - 6x + 8y - 11 = 0.	Completing the square: (x - 3)^2 + (y + 4)^2 = 9 + 16 + 11 = 36. The centre is (3, -4) and the radius is 6.	4
10	Find the equation of the tangent to the circle x^2 + y^2 = 25 at the point (3, 4).	The radius has slope 4/3, so the tangent has slope -3/4. The tangent is y - 4 = (-3/4)(x - 3), i.e. 3x + 4y = 25. Check: 3(3) + 4(4) = 25.	4
11	The line y = x + 1 meets the circle x^2 + y^2 = 25 at A and B. (a) Find the coordinates of A and B. (b) Find the length of AB.	(a) Substituting: x^2 + (x + 1)^2 = 25, so x^2 + x - 12 = 0 and x = 3 or -4, giving A(3, 4) and B(-4, -3). (b) AB = sqrt(49 + 49) = 7 sqrt 2, about 9.90.	4
12	Find the equation of the locus of points equidistant from A(0, 0) and B(6, 0), and describe the locus.	Let P(x, y) be a point on the locus. PA^2 = PB^2 gives x^2 + y^2 = (x - 6)^2 + y^2, so x^2 = x^2 - 12x + 36 and x = 3. The locus is the perpendicular bisector of AB, the vertical line x = 3.	4

Section D - Sequences, Logarithms and Growth

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
13	A geometric sequence has first term 3 and common ratio 2. (a) Find the 8th term. (b) Find the sum of the first 8 terms.	(a) T(8) = 3 x 2^(8-1) = 3 x 128 = 384. (b) S(8) = 3(2^8 - 1)/(2 - 1) = 3 x 255 = 765.	4
14	An arithmetic sequence has first term 5 and common difference 7. (a) Find the 20th term. (b) Find the sum of the first 20 terms.	(a) T(20) = 5 + (20 - 1) x 7 = 5 + 133 = 138. (b) S(20) = 20(5 + 138)/2 = 1430.	4
15	(a) Find log base 2 of 32. (b) Find log base 3 of 1/27. (c) Solve 3^(2x - 1) = 81.	(a) 2^5 = 32, so log base 2 of 32 is 5. (b) 3^(-3) = 1/27, so log base 3 of 1/27 is -3. (c) 81 = 3^4, so 2x - 1 = 4 and x = 2.5.	4
16	A city has a population of 500,000 and grows at 2.5% per year. (a) Find the population after 5 years, to the nearest integer. (b) Find the doubling time in years, correct to 2 decimal places.	(a) Population = 500,000 x 1.025^5 = 565,704. (b) 1.025^n = 2, so n = ln 2 / ln 1.025 = 28.07 years.	4

Section E - Permutations, Combinations and Probability

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
17	Three people are chosen from 8 to form a committee. (a) How many committees are possible? (b) How many contain one particular person? (c) In how many ways can 5 people stand in a row if 2 particular people must be adjacent?	(a) C(8, 3) = 56. (b) C(7, 2) = 21. (c) Treat the pair as one unit: 4! x 2 = 48.	5

18	A bag contains 5 red and 3 blue balls. Two balls are drawn at random without replacement. (a) Find the probability that both are red. (b) Find the probability that at least one is blue.	(a) P = C(5, 2)/C(8, 2) = 10/28 = 5/14, about 0.357. (b) P = 1 - 5/14 = 9/14, about 0.643.	5
19	Given P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.2. (a) Find P(A | B). (b) Determine whether A and B are independent, giving a reason.	(a) P(A | B) = P(A and B)/P(B) = 0.2/0.4 = 0.5. (b) P(A) x P(B) = 0.5 x 0.4 = 0.2 = P(A and B), so A and B are independent.	4
20	Let X follow a binomial distribution with n = 10 and p = 0.3. (a) Find P(X = 3) correct to 4 decimal places. (b) Find E(X) and the standard deviation, correct to 4 decimal places.	(a) P(X = 3) = C(10, 3)(0.3)^3(0.7)^7 = 120 x 0.027 x 0.0823543 = 0.2668. (b) E(X) = np = 3.0 and sd = sqrt(np(1 - p)) = 1.4491.	4

Section F - Normal Distribution and Statistics

Answer all questions. Show your working; marks for method are not awarded without it.

#	Question	Answer	Marks
21	Exam scores are normally distributed with mean 60 and standard deviation 8. (a) Find the standard scores of 72 and 52. (b) Given P(Z < 1.5) = 0.9332, find the percentage of scores above 72.	(a) z(72) = (72 - 60)/8 = 1.5; z(52) = (52 - 60)/8 = -1.0. (b) P(X > 72) = 1 - 0.9332 = 0.0668, i.e. 6.68%.	5
22	For the data 4, 6, 6, 8, 11, find (a) the mean, (b) the variance and (c) the standard deviation, correct to 5 decimal places.	(a) Mean = 35/5 = 7. (b) Variance = 28/5 = 5.6. (c) Standard deviation = sqrt 5.6 = 2.36643.	5
23	For the data 12, 15, 15, 18, 20, 22, find (a) the mean, (b) the median, (c) the mode and (d) the range.	(a) Mean = 102/6 = 17. (b) Median = (15 + 18)/2 = 16.5. (c) Mode = 15. (d) Range = 22 - 12 = 10.	4
24	For 5 pairs of data, sum x = 30, sum y = 50, sum xy = 340, sum x^2 = 220 and sum y^2 = 560. (a) Find the regression line of y on x. (b) Hence estimate y when x = 10.	(a) Slope b = (5 x 340 - 30 x 50)/(5 x 220 - 30^2) = 200/200 = 1.0; intercept a = (50 - 1.0 x 30)/5 = 4.0, so y = x + 4. (b) When x = 10, y is about 14.	4