S5 Maths Extended M1 Practice Paper: binomial theorem and sequences, indices and logarithms, limits and differentiation, applications of differentiation, integration and area, trapezoidal rule, binomial and normal distributions (100 marks)

Level: S5 | Subject: Maths Extended M1 | 75 minutes | 100 marks | 24 questions | answer key

Section A - Binomial Theorem and Sequences (18 marks)

Answer all questions and show your working.

#	Question	Answer	Marks
1	Expand (1 + 2x)^5 and find (a) the coefficient of x^3; (b) the sum of all the coefficients; (c) the constant term.	(a) C(5,3) x 2^3 = 10 x 8 = 80; (b) put x = 1: (1 + 2)^5 = 243; (c) constant term = 1	5
2	In the expansion of (2 + x/2)^8, find (a) the coefficient of x^5; (b) the number of terms.	(a) general term C(8,r) 2^(8-r) (x/2)^r with r = 5: C(8,5) x 2^3 x (1/2)^5 = 56 x 8/32 = 14; (b) 8 + 1 = 9 terms	5
3	An arithmetic sequence has first term 7 and common difference 5. Find (a) the value of n for which the nth term is 122; (b) the sum of the first 24 terms.	(a) 7 + 5(n - 1) = 122 gives n = 24; (b) S24 = 24/2 x (7 + 122) = 12 x 129 = 1548	4
4	A geometric sequence has first term 5 and common ratio 1/3. Find (a) the sum to infinity; (b) the 6th term, correct to 4 decimal places.	(a) S = 5 / (1 - 1/3) = 7.5; (b) 6th term = 5 x (1/3)^5 = 5/243 = 0.0206	4

Section B - Indices, Logarithms and Exponential Growth (16 marks)

Answer all questions and show your working.

#	Question	Answer	Marks
5	Solve the equation 3^(2x - 1) = 27^(x + 1).	27^(x+1) = 3^(3x+3), so 2x - 1 = 3x + 3, giving x = -4	4
6	Find (a) the value of log2 48 - log2 3; (b) the value of a if log_a 9 = 2.	(a) log2(48 / 3) = log2 16 = 4; (b) a^2 = 9 with a > 0, so a = 3	4
7	Bacteria in a dish double every 4 hours, starting from 500. Find (a) the number after 24 hours; (b) the time for the number to reach 16000 for the first time.	(a) 24 / 4 = 6, 500 x 2^6 = 32000; (b) 500 x 2^k = 16000, so 2^k = 32, k = 5, i.e. 20 hours	4
8	A radioactive substance has a half-life of 12 years and a sample of mass 80 g. Find (a) the mass after 36 years; (b) the relation between mass m and time t in years.	(a) 36 / 12 = 3, 80 x (1/2)^3 = 10 g; (b) m = 80 x 2^(-t/12) in grams	4

Section C - Limits and Differentiation (18 marks)

Answer all questions and show your working.

#	Question	Answer	Marks
9	Let y = 3x^4 - 2x^3 + 5x - 7. Find (a) dy/dx; (b) the slope of the curve at x = 2.	(a) dy/dx = 12x^3 - 6x^2 + 5; (b) at x = 2 the slope is 96 - 24 + 5 = 77	5

10	Let f(x) = (x^2 + 1)(3x - 2). Find (a) f'(x); (b) f'(1).	(a) by the product rule f'(x) = 2x(3x - 2) + 3(x^2 + 1) = 9x^2 - 4x + 3; (b) f'(1) = 8	5
11	Evaluate the limits: (a) as x tends to 3, (x^2 - 9)/(x - 3); (b) as x tends to 0, sin 5x / x.	(a) factorise the numerator to (x - 3)(x + 3), cancel to get x + 3, limit 6; (b) the limit is 5	4
12	Evaluate the limits: (a) as x tends to infinity, (4x^2 + 3x - 1)/(2x^2 - x + 5); (b) as x tends to 2, (x^3 - 8)/(x - 2).	(a) divide top and bottom by x^2, limit 4/2 = 2; (b) the derivative 3x^2 at x = 2 gives 12	4

Section D - Applications of Differentiation (16 marks)

Answer all questions and show your working.

#	Question	Answer	Marks
13	The curve is y = x^3 - 2x. Find (a) the equation of the tangent at x = 1; (b) its y-intercept.	(a) y' = 3x^2 - 2, so the slope at x = 1 is 1 and the point is (1, -1), giving y = x - 2; (b) y-intercept -2	4
14	Let y = x^3 - 3x^2 + 4. Find (a) the x values of all stationary points; (b) the nature of each and the corresponding value of y.	(a) y' = 3x(x - 2), so x = 0 or x = 2; (b) x = 0 gives a maximum with y = 4, x = 2 gives a minimum with y = 0	4
15	The radius of a sphere increases at 0.5 cm per second. When the radius is 6 cm, find the rate of increase of the volume (take pi = 3.14, 1 decimal place).	V = (4/3) pi r^3, dV/dt = 4 pi r^2 x dr/dt = 4 x 3.14 x 36 x 0.5 = 226.1 cm3 per second	4
16	A rectangular sheet has perimeter 40 cm. Find the largest area it can enclose.	let the length be x and the width 20 - x, A = x(20 - x), A' = 20 - 2x = 0 gives x = 10, so the maximum area is 100 cm2	4

Section E - Integration, Area and Trapezoidal Rule (16 marks)

Answer all questions and show your working.

#	Question	Answer	Marks
17	Evaluate the integral from 1 to 2 of (6x^2 - 4x + 3) dx.	the antiderivative is 2x^3 - 2x^2 + 3x, which is 14 at x = 2 and 3 at x = 1, so the integral is 11	4
18	Find the area of the region enclosed by the curve y = x^2 and the line y = 4.	the curves meet at x = -2 and x = 2, area = integral of (4 - x^2) = 32/3 = 10.67	4
19	Let f(x) = x^2 with values taken at x = 1, 2, 3 and 4. Find (a) the trapezoidal estimate of the integral from 1 to 4; (b) the absolute error compared with the exact value.	(a) h = 1, T = 0.5 x [1 + 16 + 2 x (4 + 9)] = 21.5; (b) the exact value is 21, so the error is 0.5	4
20	Evaluate (a) the integral from 0 to 2 of (3x^2 + 2x) dx; (b) the integral from 1 to 4 of the square root of x dx, to 3 significant figures.	(a) 12; (b) (2/3)(8 - 1) = 14/3 = 4.67	4

Section F - Probability and Statistics Applications (16 marks)

Answer all questions and show your working.

#	Question	Answer	Marks

21	Let X follow the binomial distribution with n = 10 and p = 0.3. Find (a) the mean and the variance; (b) P(X = 3), to 3 decimal places.	(a) mean 3, variance 2.1; (b) C(10,3) x 0.3^3 x 0.7^7 = 0.267	4
22	Exam scores are normally distributed with mean 60 and standard deviation 8. Given P(Z < 1) = 0.8413 and P(Z < 2) = 0.9772, find (a) P(X < 68); (b) k such that P(X < k) = 0.9772.	(a) z = (68 - 60)/8 = 1, so the probability is 0.8413; (b) z = 2 gives (k - 60)/8 = 2, so k = 76	4
23	Given P(A) = 0.6, P(B | A) = 0.5 and P(B | not A) = 0.25, find (a) P(B); (b) P(A | B).	(a) P(B) = 0.3 + 0.1 = 0.4; (b) P(A | B) = 0.3 / 0.4 = 0.75	4
24	A help desk receives 2 calls per hour on average, following a Poisson distribution. Taking e^(-2) = 0.1353, find (a) the probability of no calls in an hour; (b) the probability of at most one call in an hour, to 4 decimal places.	(a) 0.1353; (b) 3 x 0.1353 = 0.4059	4