S3 Mathematics Practice Paper (Set 1) | Percentage growth, ratio and variation, quadratic equations, mensuration, probability and statistics, coordinate geometry

S3 Mathematics | Time allowed: 75 minutes | Total: 100 marks | 24 questions

Section A: Percentages and growth

Question 1 (16 marks)

1. A jacket is marked at 640 dollars. A shop gives a 15% discount and then a further 10% discount on the discounted price. Find the final selling price. (4 marks)

> **Answer:** After the first discount: 640 x (1 - 15%) = 544 dollars. After the second discount: 544 x (1 - 10%) = 489.6 dollars, so the final selling price is 489.6 dollars.

2. Find the single equivalent discount rate of the two successive discounts. (4 marks)

> **Answer:** Equivalent discount = 1 - 0.85 x 0.90 = 0.235 = 23.5%.

3. The population of a city rises from 250 000 to 280 000. Find the percentage increase in population. (4 marks)

> **Answer:** Increase = 280 000 - 250 000 = 30 000, so percentage increase = 30 000 / 250 000 x 100% = 12%.

4. Someone claims that a 20% increase followed by a 20% decrease brings the price back to its original level. Use a calculation to show whether this claim is correct. (4 marks)

> **Answer:** Let the original price be 100 dollars. After a 20% increase it is 120 dollars, and after a 20% decrease it is 120 x 0.8 = 96 dollars, which is 4% below the original price, so the claim is wrong.

Section B: Ratio and variation

Question 2 (16 marks)

5. It is given that y varies directly as the square of x, and y = 45 when x = 3. (i) Find the constant of variation k. (ii) Find y when x = 5. (4 marks)

> **Answer:** (i) 45 = k x 3^2, so k = 5. (ii) y = 5 x 5^2 = 125.

6. It is given that z varies inversely as w, and z = 15 when w = 8. Find z when w = 12. (4 marks)

> **Answer:** zw = k, so k = 15 x 8 = 120. When w = 12, z = 120 / 12 = 10.

7. An amount of 4 200 dollars is divided between A and B in the ratio 3 : 4. Find the amount each receives. (4 marks)

> **Answer:** Total number of parts = 3 + 4 = 7, so one part = 4 200 / 7 = 600 dollars. A receives 3 x 600 = 1800 dollars and B receives 4 x 600 = 2400 dollars.

8. Compare the graphs of a direct proportion and an inverse proportion. (4 marks)

> **Answer:** The graph of a direct proportion y = kx is a straight line through the origin, and y increases as x increases. The graph of an inverse proportion z = k/w is a curve that does not pass through the origin, and z decreases as w increases.

Section C: Algebraic fractions and quadratic equations

Question 3 (18 marks)

9. Simplify (x^2 - 9) / (x^2 + x - 12) and state the restrictions on x. (5 marks)

> **Answer:** x^2 - 9 = (x - 3)(x + 3) and x^2 + x - 12 = (x + 4)(x - 3), so the fraction simplifies to (x + 3) / (x + 4), where x is not 3 and x is not -4.

10. Solve the equation 2x^2 + 3x - 2 = 0. (5 marks)

> **Answer:** (2x - 1)(x + 2) = 0, so x = 1/2 or x = -2.

11. Solve the equation x^2 + 6x - 7 = 0 by completing the square. (4 marks)

> **Answer:** x^2 + 6x = 7, and adding 9 to both sides gives x^2 + 6x + 9 = 16, that is (x + 3)^2 = 16. Hence x + 3 = 4 or -4, so x = 1 or x = -7.

12. The quadratic equation x^2 + kx + 9 = 0 has a repeated root. Find the possible values of k. (4 marks)

> **Answer:** The discriminant is k^2 - 4 x 1 x 9 = k^2 - 36. A repeated root means the discriminant is zero, so k^2 = 36 and k = 6 or k = -6.

Section D: Mensuration

Question 4 (18 marks)

13. A cylinder has radius 7 cm and height 20 cm. Find its volume in terms of pi, correct to 1 decimal place. (4 marks)

> **Answer:** Volume = pi x 7^2 x 20 = 980 pi = 3078.8 cm^3.

14. A cone has radius 7 cm and height 24 cm. Find its volume in terms of pi, correct to 1 decimal place. (4 marks)

> **Answer:** Volume = (1/3) x pi x 7^2 x 24 = 392 pi = 1231.5 cm^3.

15. A sphere has radius 6 cm. Find its surface area and its volume in terms of pi, correct to 1 decimal place. (5 marks)

> **Answer:** Surface area = 4 x pi x 6^2 = 144 pi = 452.4 cm^2. Volume = (4/3) x pi x 6^3 = 288 pi = 904.8 cm^3.

16. The cylinder in part (a) is melted and recast into solid spheres of radius 7 cm. Find the greatest number of spheres that can be made. (5 marks)

> **Answer:** Volume of one sphere = (4/3) x pi x 7^3 = 457.3 pi cm^3, about 1436.8 cm^3. Since 980 pi / 457.3 pi is about 2.14, at most 2 spheres can be made.

Section E: Probability and statistics

Question 5 (16 marks)

17. A bag contains 5 red, 3 blue and 2 green balls. A ball is drawn at random. Find the probability that it is not red. (4 marks)

> **Answer:** Total number of balls = 5 + 3 + 2 = 10, and 5 of them are not red, so the probability is 5/10 = 0.5.

18. Two balls are drawn one after another without replacement. Find the probability that both balls are red. (4 marks)

> **Answer:** P(both red) = (5/10) x (4/9) = 20/90 = 2/9 = 0.222.

19. For the data 4, 7, 7, 9, 13, find the mean, the median, the mode and the range. (4 marks)

> **Answer:** Mean = (4 + 7 + 7 + 9 + 13) / 5 = 8. Median = 7. Mode = 7. Range = 13 - 4 = 9.

20. Find the standard deviation of the data above, correct to 3 significant figures. (4 marks)

> **Answer:** Sum of squared deviations = 16 + 1 + 1 + 1 + 25 = 44, so the variance is 44 / 5 = 8.8 and the standard deviation is the square root of 8.8, that is 2.97.

Section F: Coordinate geometry

Question 6 (16 marks)

21. A(1, 2) and B(5, 10) are given. Find the length of AB in surd form, correct to 2 decimal places. (4 marks)

> **Answer:** AB^2 = (5 - 1)^2 + (10 - 2)^2 = 16 + 64 = 80, so AB = sqrt(80) = 4 sqrt(5) = 8.94 units.

22. Find the slope of the line AB. (4 marks)

> **Answer:** Slope = (10 - 2) / (5 - 1) = 8 / 4 = 2.

23. Find the equation of the perpendicular bisector of AB, giving the answer in general form. (4 marks)

> **Answer:** Midpoint = ((1 + 5)/2, (2 + 10)/2) = (3, 6). The slope of AB is 2, so the perpendicular bisector has slope -1/2. Its equation is y - 6 = -1/2 (x - 3), that is x + 2y - 15 = 0.

24. The equation of a line L is 3x + 4y - 12 = 0. Find the slope, the x-intercept and the y-intercept of L. (4 marks)

> **Answer:** From 3x + 4y = 12 we get y = -3/4 x + 3, so the slope is -3/4 and the y-intercept is 3. Putting y = 0 gives x = 4, so the x-intercept is 4.