S4 Mathematics Extended Part Module 2 Practice Paper (Set 1) | Mathematical induction, binomial theorem, trigonometry, limits and differentiation, integration, matrices and vectors

S4 Mathematics Extended Part Module 2 | Time allowed: 75 minutes | Total: 100 marks | 24 questions

Section A: Mathematical induction and the binomial theorem

Question 1 (16 marks)

1. Prove by mathematical induction that for all positive integers n, 1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6. (4 marks)

2. Using the formula in the previous part, evaluate 10^2 + 11^2 + ... + 15^2. (4 marks)

3. Find the constant term in the expansion of (2x - 3/x)^6. (4 marks)

4. Find the coefficient of x^4 in the expansion of (2x - 3/x)^6. (4 marks)

Section B: Trigonometry

Question 2 (16 marks)

5. Without using a calculator, find the exact value of sin 15 degrees. (4 marks)

6. Given that A and B are acute angles with sin A = 3/5 and cos B = 5/13, find sin(A + B). (4 marks)

7. Solve 2sin^2 x + sin x - 1 = 0 for 0 deg <= x <= 360 deg. (4 marks)

8. Prove the identity (1 - cos 2theta) / sin 2theta = tan theta. (4 marks)

Section C: Limits and differentiation

Question 3 (16 marks)

9. Evaluate lim(x->3) (x^2 - 9)/(x - 3). (4 marks)

10. Let y = (3x^2 + 1)^4. Find the value of dy/dx when x = 1. (4 marks)

11. Find the equation of the tangent to the curve y = x^3 - 3x at x = 2. (4 marks)

12. Evaluate lim(x->0) (1 - cos x)/x^2. (4 marks)

Section D: Integration

Question 4 (16 marks)

13. Evaluate the definite integral of (2x + 1) from 0 to 3. (4 marks)

14. Find the area of the region bounded by the line y = 2x and the curve y = x^2. (4 marks)

15. Evaluate the integral of 2x/(x^2 + 1) from 0 to 1. (4 marks)

16. The curve y = x^2 (0 <= x <= 2) is rotated through one revolution about the x-axis. Find the volume of the solid formed. (4 marks)

Section E: Matrices and linear transformations

Question 5 (16 marks)

17. Let A = [[3, 1], [2, 4]]. Find det A and the inverse of A. (4 marks)

18. Solve the simultaneous equations 3x + y = 7 and 2x + 4y = 18. (4 marks)

19. Write down the matrix for a rotation of 90 degrees anticlockwise, and find the image of the point (3, -2) under this transformation. (4 marks)

20. Let B = [[2, -1], [5, 3]]. Find det B and det(AB), where A is the matrix above. (4 marks)

Section F: Vectors

Question 6 (20 marks)

21. Let a = (2, -1, 3) and b = (1, 4, -2). Find the scalar product a.b and the magnitude of a (correct to 3 decimal places). (5 marks)

22. Find the angle between a and b, correct to 0.1 degree. (5 marks)

23. Find the vector product a x b, and hence the area of the parallelogram with adjacent sides a and b. (5 marks)

24. Find the vector projection of b onto a. (5 marks)
