S4 Mathematics Extended Part Module 1 Practice Paper: Binomial Theorem, Exponential and Logarithmic Equations, Limits and Derivatives, Differentiation Rules with Tangents and Normals, Extrema and Applications, Definite Integrals and the Trapezoidal Rule (100 marks)

S4 | Mathematics Extended Part Module 1 | Time: 75 minutes | Total: 100 marks | 24 questions

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Part 1: Binomial Theorem (16 marks)

Show the expansion or binomial coefficient method.

1. Find the coefficient of x^3 in the expansion of (1 + 2x)^5. [4]

2. Find the coefficient of x^2 in the expansion of (2 - x)^6. [4]

3. Find the coefficient of x^2 in (1 + 3x)(1 + x)^7. [4]

4. Let n be a positive integer. If the coefficient of x^2 in (1 + x)^n is 45, find n. [4]

Part 2: Indices and Logarithmic Equations (16 marks)

Give exact answers; use logarithms where the answer is not an integer.

5. Solve 3^(2x-1) = 27. [4]

6. Solve log2 x + log2 (x - 2) = 3. [4]

7. Solve e^(2x) - 5e^x + 6 = 0, giving answers in logarithmic form. [4]

8. Simplify log_a 8 + log_a 2 - log_a 4, where a > 0 and a is not 1. [4]

Part 3: Limits and Differentiation from First Principles (18 marks)

Do not substitute directly; show valid limiting arguments.

9. Find lim (x→3) (x^2 - 9)/(x - 3). [4]

10. Find lim (x→0) (sqrt(1 + x) - 1)/x. [4]

11. Let f(x) = x^2 + 3x. Use first principles to find f'(x), then evaluate f'(2). [5]

12. Find lim (x→∞) (3x^2 + 5x)/(2x^2 - 1). [5]

Part 4: Differentiation Rules, Tangents and Normals (18 marks)

Write the derivative first, then substitute.

13. Given y = x^3 - 4x, find dy/dx at x = 2. [5]

14. Given y = (2x + 1)^5, find dy/dx at x = 1. [5]

15. Find the equation of the tangent to y = x^2 at (3, 9). [4]

16. Find the equation of the normal to y = x^2 at (3, 9). [4]

Part 5: Extrema and Related Rates (18 marks)

Justify extrema with the first derivative.

17. Find the local maximum and local minimum of y = x^3 - 3x^2 + 4. [4]

18. A rectangle has perimeter 40 cm. Find its maximum area. [4]

19. A sphere has volume V = (4/3)πr^3. When r = 3 cm the volume increases at 36π cm^3 per second. Find the rate of increase of the radius. [5]

20. Given y = x + 4/x for x > 0, find the minimum value of y and the corresponding x. [5]

Part 6: Definite Integrals and the Trapezoidal Rule (14 marks)

Give numerical answers to three decimal places where needed.

21. Evaluate the definite integral of 3x^2 + 2x from x = 1 to x = 2. [4]

22. Evaluate the definite integral of 1/x from x = 1 to x = e. [4]

23. Use the trapezoidal rule with h = 0.5 and five ordinates to estimate the integral of x^2 from 0 to 2, and state whether the estimate is too large or too small. [3]

24. Find the area between the curve y = x^2 and the x-axis from x = 0 to x = 3. [3]
