S4 Mathematics Practice Paper (Set 2): completing the square and discriminant, exponential and logarithmic applications, trigonometric ratios and 3-D measurement, equation

Level: S4 | Subject: Mathematics | Time: 75 min | Total: 100 marks | Items: 24

Question 1 Quadratic functions and the discriminant

Show all working in this question.

1. Let $f(x) = -2x^2 + 12x - 7$. By completing the square, find the vertex and the maximum value of $f(x)$. (4 marks)

Answer: $f(x) = -2(x - 3)^2 + 11$, vertex $(3, 11)$, maximum value 11.

2. Solve $f(x) = 0$, giving answers in surd form, and state the nature of the roots. (4 marks)

Answer: $2x^2 - 12x + 7 = 0$, discriminant $= 88 > 0$, two distinct real roots; $x = 3 \pm \sqrt{22}/2 \approx 5.35$ or 0.65.

3. Let $g(x) = 2x^2 - 12x + k$. If $g(x) = 0$ has two distinct real roots, find the range of k. (5 marks)

Answer: Discriminant $= 144 - 8k > 0$, so $k < 18$.

4. A ball is thrown so that its height is $h = -5t^2 + 20t + 1.5$ (metres, t seconds). Find the maximum height and the time it hits the ground (3 d.p.). (5 marks)

Answer: $h = -5(t - 2)^2 + 21.5$, maximum 21.5 m at $t = 2$; hits ground at $t \approx 4.074$ s.

Question 2 Exponentials and logarithms

Give answers to 3 significant figures unless stated.

5. Solve $3^{(2x-1)} = 27^{(x-2)}$. (3 marks)

Answer: $27^{(x-2)} = 3^{(3x-6)}$, so $2x - 1 = 3x - 6$, $x = 5$.

6. Simplify and evaluate $\log_2 48 - \log_2 3 + \log_2(1/2)$. (4 marks)

Answer: $= \log_2(48 \div 3 \times 1/2) = \log_2 8 = 3$.

7. A radioactive substance has mass $M = 80(0.5)^{(t/12)}$ grams after t hours. Find t when $M = 10$ g. (5 marks)

Answer: $(0.5)^{(t/12)} = (0.5)^3$, so $t/12 = 3$, $t = 36$ hours.

8. \$5000 is deposited at 4% p.a. compounded quarterly. Find the least number of years for the amount to exceed \$8000. (4 marks)

Answer: $5000(1.01)^{(4n)} > 8000$ gives $4n > 47.24$, $n > 11.81$, so 12 years.

Question 3 Trigonometry and 3-D measurement

Give units in every answer.

9. In triangle ABC, AB = 8 cm, AC = 11 cm and ∠A = 55°. Find BC. (4 marks)

Answer: $BC^2 = 185 - 176\cos 55° = 84.05$, so BC ≈ 9.17 cm.

10. Find the area of triangle ABC. (4 marks)

Answer: Area = ½(8)(11)sin55° ≈ 36.0 cm².

11. A vertical tower casts a 12 m shadow on level ground when the angle of elevation of the sun is 38°. Find the height of the tower. (4 marks)

Answer: Height = 12 tan38° ≈ 9.38 m.

12. A cuboid measures 9 cm × 6 cm × 5 cm. Find the length of the space diagonal and the angle it makes with the base. (5 marks)

Answer: Diagonal = √142 ≈ 11.9 cm; base diagonal √117, angle ≈ 24.8°.

Question 4 Coordinate geometry and the circle

Given A(1, -2) and B(7, 6).

13. Find the length of AB and the coordinates of its midpoint. (4 marks)

Answer: AB = 10; midpoint (4, 2).

14. Find the equation of the circle with AB as a diameter. (4 marks)

Answer: Centre (4, 2), radius 5: $(x - 4)^2 + (y - 2)^2 = 25$.

15. Find the slope of the tangent to the circle at B. (4 marks)

Answer: Radius slope = 4/3, so tangent slope = -3/4.

16. The line y = 2x - 3 cuts the circle at two points. Find the length of the chord. (5 marks)

Answer: Distance from centre = 3/√5 ≈ 1.342; chord = 2√(25 - 1.8) ≈ 9.63.

Question 5 Probability and expected value

A survey of 40 students: boys — bus 6, MTR 10, walk 4; girls — bus 6, MTR 8, walk 6.

17. A student is chosen at random. Find the probability of travelling by MTR. (4 marks)

Answer: (10 + 8)/40 = 0.45.

18. Given that the chosen student travels by bus, find the probability that the student is a girl. (4 marks)

Answer: 6/12 = 0.5.

19. Three students are chosen from 20 boys and 20 girls. Find the number of ways of choosing 2 boys and 1 girl. (4 marks)

Answer: C(20,2) × 20 = 3800.

20. A game costs $8 per play, paying $20 with probability 0.3, $5 with probability 0.5, nothing with probability 0.2. Find the expected payout and the expected profit per play. (4 marks)

Answer: Expected payout = 6.5; expected profit = 8 - 6.5 = 1.5.

Question 6 Statistics

Daily study time (minutes) of 12 students: 42, 55, 38, 61, 47, 52, 45, 58, 40, 49, 53, 50.

21. Find the mean and the median. (4 marks)

Answer: Mean = 590/12 ≈ 49.2 min; median = 49.5 min.

22. Find the standard deviation, dividing by 12, correct to 3 d.p. (5 marks)

Answer: $\Sigma(x - \bar{x})^2 = 557.67$; variance = 46.47; s.d. ≈ 6.817 min.

23. If every student studies 5 more minutes, state the new mean and standard deviation, and explain. (4 marks)

Answer: Mean becomes 54.2 (plus 5); s.d. unchanged at 6.817 since shifting adds a constant.

24. Class A (25 students) averages 48 min, class B (15 students) averages 56 min. Find the combined mean. (3 marks)

Answer: 2040/40 = 51 min.
