S4 Mathematics Practice Paper (Set 2): completing the square and discriminant, exponential and logarithmic applications, trigonometric ratios and 3-D measurement, equation

Level: S4 | Subject: Mathematics | Time: 75 min | Total: 100 marks | Items: 24

Question 1 Quadratic functions and the discriminant

Show all working in this question.

1. Let $f(x) = -2x^2 + 12x - 7$. By completing the square, find the vertex and the maximum value of $f(x)$. (4 marks)

2. Solve $f(x) = 0$, giving answers in surd form, and state the nature of the roots. (4 marks)

3. Let $g(x) = 2x^2 - 12x + k$. If $g(x) = 0$ has two distinct real roots, find the range of k. (5 marks)

4. A ball is thrown so that its height is $h = -5t^2 + 20t + 1.5$ (metres, t seconds). Find the maximum height and the time it hits the ground (3 d.p.). (5 marks)

Question 2 Exponentials and logarithms

Give answers to 3 significant figures unless stated.

5. Solve $3^{(2x-1)} = 27^{(x-2)}$. (3 marks)

6. Simplify and evaluate $\log_2 48 - \log_2 3 + \log_2 (1/2)$. (4 marks)

7. A radioactive substance has mass $M = 80(0.5)^{(t/12)}$ grams after t hours. Find t when $M = 10$ g. (5 marks)

8. \$5000 is deposited at 4% p.a. compounded quarterly. Find the least number of years for the amount to exceed \$8000. (4 marks)

Question 3 Trigonometry and 3-D measurement

Give units in every answer.

9. In triangle ABC, AB = 8 cm, AC = 11 cm and $\angle A = 55°$. Find BC. (4 marks)

10. Find the area of triangle ABC. (4 marks)

11. A vertical tower casts a 12 m shadow on level ground when the angle of elevation of the sun is 38°. Find the height of the tower. (4 marks)

12. A cuboid measures 9 cm × 6 cm × 5 cm. Find the length of the space diagonal and the angle it makes with the base. (5 marks)

Question 4 Coordinate geometry and the circle

Given A(1, -2) and B(7, 6).

13. Find the length of AB and the coordinates of its midpoint. (4 marks)

14. Find the equation of the circle with AB as a diameter. (4 marks)

15. Find the slope of the tangent to the circle at B. (4 marks)

16. The line y = 2x - 3 cuts the circle at two points. Find the length of the chord. (5 marks)

Question 5 Probability and expected value

A survey of 40 students: boys — bus 6, MTR 10, walk 4; girls — bus 6, MTR 8, walk 6.

17. A student is chosen at random. Find the probability of travelling by MTR. (4 marks)

18. Given that the chosen student travels by bus, find the probability that the student is a girl. (4 marks)

19. Three students are chosen from 20 boys and 20 girls. Find the number of ways of choosing 2 boys and 1 girl. (4 marks)

20. A game costs $8 per play, paying $20 with probability 0.3, $5 with probability 0.5, nothing with probability 0.2. Find the expected payout and the expected profit per play. (4 marks)

Question 6 Statistics

Daily study time (minutes) of 12 students: 42, 55, 38, 61, 47, 52, 45, 58, 40, 49, 53, 50.

21. Find the mean and the median. (4 marks)

22. Find the standard deviation, dividing by 12, correct to 3 d.p. (5 marks)

23. If every student studies 5 more minutes, state the new mean and standard deviation, and explain. (4 marks)

24. Class A (25 students) averages 48 min, class B (15 students) averages 56 min. Find the combined mean. (3 marks)
