S4 Mathematics Practice Paper 1 | Indices, sequences, quadratics, circles, probability

Senior Secondary Mathematics

S4 Mathematics practice paper: indices, logarithms, sequences, quadratic functions, circle properties, probability and compound interest. 100 marks with full answers.

1. Indices and logarithms (16 marks)

Parts (a) to (c) are algebraic and need working; part (d) is an application.

#	Question	Answer and marking notes
1	(a) Evaluate log_3 81 + log_2 32.	Since 3^4 = 81, log_3 81 = 4; since 2^5 = 32, log_2 32 = 5; the sum is 9.
2	(b) Solve 2^(x+1) = 64.	2^(x+1) = 2^6, so x + 1 = 6 and x = 5.
3	(c) Solve log x + log(x - 3) = 1 for x > 3.	log[x(x - 3)] = 1 gives x(x - 3) = 10, so x^2 - 3x - 10 = 0 and (x - 5)(x + 2) = 0; since x > 3, x = 5.
4	(d) An investment grows at 6% per year compounded yearly. Use logarithms to find, to the nearest whole year, the time for the money to double.	1.06^n = 2, so n = log 2 / log 1.06 = 11.896, i.e. about 12 years.

2. Arithmetic and geometric sequences (18 marks)

An arithmetic sequence has first term 7 and common difference 4; a geometric sequence has first term 3 and common ratio 2.

#	Question	Answer and marking notes
5	(a) Find the 15th term of the arithmetic sequence.	T(15) = 7 + (15 - 1) x 4 = 63.
6	(b) Find the sum of the first 20 terms.	S(20) = 20/2 x [2×7 + 19×4] = 10 x 90 = 900.
7	(c) Find the 8th term and the sum of the first 8 terms of the geometric sequence.	T(8) = 3 x 2^7 = 384; S(8) = 3(2^8 - 1)/(2 - 1) = 765.
8	(d) Another geometric sequence has first term 8 and common ratio 0.6. Find its sum to infinity.	S = a / (1 - r) = 8 / (1 - 0.6) = 20.

3. Quadratic equations and functions (16 marks)

Consider the equation x^2 - 7x + 12 = 0 and the function y = x^2 - 6x + 5.

#	Question	Answer and marking notes
9	(a) Solve x^2 - 7x + 12 = 0.	(x - 3)(x - 4) = 0, so x = 3 or x = 4.
10	(b) If x^2 + kx + 9 = 0 has a repeated root, find all possible values of k.	Discriminant = k^2 - 4x1x9 = 0, so k^2 = 36 and k = 6 or k = -6.

#	Question	Answer and marking notes
11	(c) Find the coordinates of the vertex of y = x^2 - 6x + 5.	x = -b/2a = 6/2 = 3; y = 9 - 18 + 5 = -4; the vertex is (3, -4).
12	(d) Find where the graph cuts the x-axis and state the direction of opening.	x^2 - 6x + 5 = 0 gives (x - 1)(x - 5) = 0, so the intercepts are (1, 0) and (5, 0); the coefficient of x^2 is positive, so it opens upwards.

4. Properties of circles (18 marks)

A circle has centre O and radius 10 cm; chord AB is 6 cm from O. Another circle has centre O(0, 0) and radius 5, and the point P(13, 0) lies outside it.

#	Question	Answer and marking notes
13	(a) Find the length of chord AB.	Half the chord = sqrt(10^2 - 6^2) = 8, so AB = 2 x 8 = 16 cm.
14	(b) In a cyclic quadrilateral ABCD, angle A = 72 degrees. Find angle C.	Opposite angles of a cyclic quadrilateral are supplementary, so angle C = 180 - 72 = 108 degrees.
15	(c) A tangent from P touches the circle at T. Find PT.	OP = 13 and OT = 5; OT is perpendicular to PT, so PT = sqrt(13^2 - 5^2) = 12.
16	(d) Find the arc length for radius 12 cm and angle 150 degrees, in terms of pi and to 1 d.p.	Arc length = 12 x (150/360) x 2pi = 10pi = 31.4 cm.

5. Permutations, combinations and probability (16 marks)

A group has 5 boys and 4 girls; a fair die is rolled twice.

#	Question	Answer and marking notes
17	(a) How many ways are there to choose 3 people from the group?	C(9, 3) = 9x8x7 / (3x2x1) = 84 ways.
18	(b) How many of these choices contain at least one girl?	All-boy choices are C(5, 3) = 10, so at least one girl gives 84 - 10 = 74 ways.
19	(c) Find the probability that the sum of two rolls is 7 (as a fraction and to 3 d.p.).	There are 6 outcomes with sum 7, so the probability is 6/36 = 1/6 = 0.167.
20	(d) From A, B, C, D, E, how many arrangements give 3 people different posts in order?	P(5, 3) = 5x4x3 = 60 arrangements.

6. Percentages, compound interest and discounts (16 marks)

A principal of 50,000 is invested at 4% per year for 3 years. An item is marked at 1,200 and is discounted by 20% and then a further 10%.

#	Question	Answer and marking notes
21	(a) Find the amount after 3 years with yearly compounding.	A = 50,000 x 1.04^3 = 56243.2.
22	(b) What would the simple interest be for 3 years?	I = 50,000 x 4% x 3 = 6000.
23	(c) How much more interest does compounding give?	Compound interest is 6243.2 and simple interest is 6000, so compounding gives 243.2 more.
24	(d) Find the final selling price of the item.	1,200 x 0.8 x 0.9 = 864.