S4 Mathematics Module 1 Practice Paper 1 | Binomial, log equations, induction, series

Senior Secondary Mathematics (Module 1)

S4 Mathematics Module 1 practice paper with worked answers: binomial coefficients, log equations, induction, series and graph transformations.

1. Binomial expansion and coefficients (18 marks)

Find the coefficients only; no need to expand fully.

#	Question	Answer and marking notes
1	(a) Find the coefficient of x^3 in the expansion of (1 + 2x)^5.	The general term is C(5, r)(2x)^r; with r = 3 the coefficient is C(5, 3) x 2^3 = 10 x 8 = 80.
2	(b) Find the coefficient of x^2 in the expansion of (2 - x)^6.	General term C(6, r) 2^(6-r) (-x)^r; with r = 2 the coefficient is C(6, 2) x 2^4 x 1 = 15 x 16 = 240.
3	(c) Find the sum of the constant term and the coefficient of x in the expansion of (1 + 3x)^7.	Constant term = 1; coefficient of x = C(7, 1) x 3 = 21; sum = 1 + 21 = 22.
4	(d) Find the sum of all coefficients in the expansion of (1 + x)^10.	Put x = 1, so the sum of coefficients is (1 + 1)^10 = 2^10 = 1024.

2. Exponential and logarithmic equations (16 marks)

Check the domain of the logarithm, and give a whole number of years in (d).

#	Question	Answer and marking notes
5	(a) Solve 2^(x+1) = 32.	Since 32 = 2^5, x + 1 = 5 and x = 4.
6	(b) Solve log base 3 of x plus log base 3 of (x - 2) equals 1.	Combine to get log base 3 of x(x - 2) = 1, so x(x - 2) = 3, giving x^2 - 2x - 3 = 0 and (x - 3)(x + 1) = 0. So x = 3 or x = -1; since both x and x - 2 must be positive, x = -1 is rejected and x = 3.
7	(c) Simplify log 8 divided by log 2 plus log base 5 of 25.	log 8 / log 2 = log base 2 of 8 = 3, and log base 5 of 25 = 2, so the total is 3 + 2 = 5.
8	(d) An amount grows at 6% per year, compounded yearly. Find the least whole number of years needed for the amount to double.	Solve 1.06^n = 2, so n = log 2 / log 1.06 = 11.90. After 11 years the factor is only 1.898, below 2, so the least whole number of years is 12 (1.06^12 = 2.012).

3. Mathematical induction (18 marks)

Write the basis step, the induction assumption and the induction step in order.

#	Question	Answer and marking notes
9	(a) For the statement 1 + 3 + 5 + … + (2n - 1) = n^2, write the basis step for n = 1.	When n = 1, the left side has the single term 2(1) - 1 = 1 and the right side is 1^2 = 1, so the two sides agree and the statement holds for n = 1.
10	(b) From (a), assume the statement holds for n = k and write the step leading to n = k + 1.	Assume 1 + 3 + … + (2k - 1) = k^2. Adding the (k + 1)th term gives k^2 + [2(k + 1) - 1] = k^2 + 2k + 1 = (k + 1)^2, which is the right side for n = k + 1. So the statement holds for n = k + 1, and by the principle of induction it holds for all positive integers n.
11	(c) Use the formula to check the sum 1 + 3 + 5 + … + 19.	The last term is 2n - 1 = 19, so n = 10 and the sum is 10^2 = 100.
12	(d) Find the sum 1 + 3 + 5 + … + 99.	Here 2n - 1 = 99, so n = 50 and the sum is 50^2 = 2500.

4. Sequences and series (16 marks)

The infinite geometric sum applies only when the common ratio has absolute value below 1.

#	Question	Answer and marking notes
13	(a) An arithmetic sequence has first term 5 and common difference 4. Find the 12th term.	a12 = 5 + (12 - 1) x 4 = 5 + 44 = 49.
14	(b) From (a), find the sum of the first 12 terms.	S12 = 12 / 2 x (5 + 49) = 6 x 54 = 324.
15	(c) A geometric sequence has first term 3 and common ratio 2. Find the sum of the first 8 terms.	S8 = 3 x (2^8 - 1) / (2 - 1) = 3 x 255 = 765.
16	(d) An infinite geometric sequence has first term 8 and common ratio 1/4. Find its sum to infinity to 2 decimal places.	Since the ratio 1/4 has modulus below 1, S = 8 / (1 - 1/4) = 8 / 0.75 = 32/3 = 10.67.

5. Transformations of graphs (16 marks)

Horizontal shifts change x; vertical shifts change the whole function value.

#	Question	Answer and marking notes
17	(a) The graph of y = f(x) is shifted 3 units to the right and then 2 units up. Write the new function.	A shift 3 units right replaces x by x - 3, and a shift 2 units up adds 2 to the value, so the new function is y = f(x - 3) + 2.
18	(b) From (a), if the original graph is y = x^2, find the vertex of the new graph.	y = (x - 3)^2 + 2, so the vertex is (3, 2).
19	(c) The graph of y = f(x) is stretched by a factor of 2 parallel to the y-axis and then reflected in the x-axis. Write the new function.	A stretch of factor 2 parallel to the y-axis gives y = 2f(x); reflecting in the x-axis gives y = -2f(x).

#	Question	Answer and marking notes
20	(d) After the graph of y = 2^x is shifted 2 units to the left, at which point does it cut the y-axis?	The new function is y = 2^(x + 2). Putting x = 0 gives y = 2^2 = 4, so the point is (0, 4).

6. Modelling with functions (16 marks)

State the domain in your model; in (d) round to the nearest thousand.

#	Question	Answer and marking notes
21	(a) A rectangle has perimeter 40 m. Let its width be x m. Express the area A in m² as a function of x and state the domain.	Length plus width is 20, so length = 20 - x and A = x(20 - x) = 20x - x^2, with x between 0 and 20.
22	(b) From (a), find the greatest possible area.	A = -(x - 10)^2 + 100, so A is greatest when x = 10, giving a maximum area of 100 m² (a 10 m by 10 m square).
23	(c) A spherical balloon is inflated from radius 3 cm to radius 6 cm. By what factor does its volume increase?	Volume varies as the cube of the radius, so the factor is (6 / 3)^3 = 2^3 = 8, i.e. 8 times the original volume.
24	(d) A city population is modelled by P(t) = 500 000 x 1.02^t, where t is in years. Find the population at t = 10, correct to the nearest thousand.	1.02^10 = 1.21899; P(10) = 500 000 x 1.21899 = 609 497, which is 609 000 to the nearest thousand.