S4 Mathematics Module 2 Practice Paper 1 | Trigonometry, logarithms, calculus, matrices, vectors

Senior Secondary Mathematics (Module 2)

S4 Mathematics Module 2 practice paper with worked answers: compound angles, logarithms, limits and derivatives, definite integrals, matrices and vectors.

1. Trigonometric identities and compound angles (16 marks)

The cosine is negative in the second quadrant; give the exact value in surd form.

#	Question	Answer and marking notes
1	(a) Given sin θ = 3/5 and θ in the second quadrant, find cos θ.	cos squared θ = 1 - 9/25 = 16/25, so cos θ = ±4/5; cosine is negative in the second quadrant, hence cos θ = -4/5.
2	(b) Use a compound angle formula to find the exact value of sin 75 degrees.	sin 75 = sin(45 + 30) = sin 45 cos 30 + cos 45 sin 30 = (root 2 / 2)(root 3 / 2) + (root 2 / 2)(1/2) = (root 6 + root 2)/4, about 0.9659.
3	(c) Simplify sin 2x cos x + cos 2x sin x.	Using sin(A + B) = sin A cos B + cos A sin B with A = 2x and B = x gives sin 3x.
4	(d) Solve 2 sin squared x - sin x - 1 = 0 for 0 to 360 degrees.	Let u = sin x, then 2u squared - u - 1 = 0, (2u + 1)(u - 1) = 0, so u = -1/2 or u = 1. sin x = 1 gives x = 90; sin x = -1/2 gives x = 210 or 330. So x = 90, 210, 330 degrees.

2. Logarithmic and exponential equations (16 marks)

Check the domain after solving; give answers in (b) to 3 decimal places.

#	Question	Answer and marking notes
5	(a) Simplify log base 2 of 48 minus log base 2 of 3.	log base 2 of 48 minus log base 2 of 3 equals log base 2 of (48/3) = log base 2 of 16 = 4.
6	(b) Solve e to the power 2x minus 5 e to the power x plus 6 equals 0, answers to 3 decimal places.	Let u = e to the x, then u squared - 5u + 6 = 0, (u - 2)(u - 3) = 0, so u = 2 or 3, giving x = ln 2 about 0.693 or x = ln 3 about 1.099.
7	(c) Solve 3 to the power (2x - 1) equals 27.	Since 27 = 3 cubed, 2x - 1 = 3 and x = 2.
8	(d) Solve ln x + ln(x - 3) = ln 10.	Combining gives ln of x(x - 3) equals ln 10, so x squared - 3x - 10 = 0, (x - 5)(x + 2) = 0, giving x = 5 or -2. Since x and x - 3 must be positive, x = -2 is rejected and x = 5.

3. Limits and differentiation (18 marks)

Factorise before cancelling the denominator; use the product rule in (b).

#	Question	Answer and marking notes
9	(a) Find the limit as x tends to 3 of (x squared - 9)/(x - 3).	x squared - 9 = (x - 3)(x + 3); cancelling (x - 3) leaves x + 3, which is 6 at x = 3.
10	(b) Differentiate 3 x squared ln x with respect to x.	By the product rule the derivative is 6x ln x + 3 x squared times 1/x, that is 6x ln x + 3x.
11	(c) Find the gradient of the tangent to y = x cubed - 3x at x = 1.	dy/dx = 3 x squared - 3; at x = 1 this is 3 - 3 = 0, so the tangent there is horizontal.
12	(d) Given y = (2x + 1) to the power 5, find dy/dx at x = 0.	dy/dx = 5(2x + 1) to the power 4 times 2 = 10(2x + 1) to the power 4; at x = 0 this is 10 x 1 = 10.

4. Integration and area (16 marks)

Write the antiderivative first for definite integrals; use the substitution u = x squared in (c).

#	Question	Answer and marking notes
13	(a) Find the integral of 6x with respect to x.	The integral of 6x is 3 x squared plus a constant.
14	(b) Evaluate the integral from 1 to 2 of (2x + 1) dx.	The antiderivative is x squared + x; substituting the limits gives (4 + 2) - (1 + 1) = 6 - 2 = 4.
15	(c) Evaluate the integral from 0 to 1 of x times e to the x squared dx, to 3 decimal places.	Put u = x squared, du = 2x dx, so the integral is one half times (e - 1), about 0.859.
16	(d) Find the area bounded by y = x squared, the x-axis and the lines x = 0 and x = 3.	The area is the integral from 0 to 3 of x squared, which is 27/3 = 9 square units.

5. Matrices and determinants (18 marks)

The inverse is 1 over det A times the adjugate; check (c) by substitution.

#	Question	Answer and marking notes
17	(a) Let A be the matrix with rows (3, 4) and (1, 2). Find det A.	det A = 3 x 2 - 4 x 1 = 6 - 4 = 2.
18	(b) From (a), find the inverse of A.	The inverse of A is one half times the matrix with rows (2, -4) and (-1, 3).
19	(c) Use the inverse matrix to solve 3x + 4y = 10 and x + 2y = 4.	Writing the system as A times (x, y) equals (10, 4), the solution is one half times (20 - 16, -10 + 12) = (2, 1). So x = 2 and y = 1, which checks in 3(2) + 4(1) = 10.
20	(d) Let B be the matrix with rows (1, 2) and (3, 4). Find B squared.	Squaring B gives the matrix with rows (1 + 6, 2 + 8) and (3 + 12, 6 + 16), that is rows (7, 10) and (15, 22).

6. Vectors and applications (16 marks)

Use cos theta = (a dot b) over the product of the lengths; find the unit vector first in (d).

#	Question	Answer and marking notes
21	(a) Let a be the vector (3, 4). Find its length.	The length is the square root of 9 + 16, that is 5.
22	(b) Let b be the vector (1, -2). Find the dot product of a and b.	The dot product is 3 x 1 + 4 x (-2) = 3 - 8 = -5.
23	(c) From (a) and (b), find the angle between a and b, to 1 decimal place.	The length of b is about 2.2361, so cos theta = -5 / (5 x 2.2361) = -0.4472 and theta is about 116.6 degrees.
24	(d) Point A is (1, 2). Moving 10 units from A in the direction of the vector (4, 3) reaches point P. Find the coordinates of P.	The vector (4, 3) has length 5, so its unit vector is (0.8, 0.6); 10 units gives a displacement of (8, 6). Hence P is (9, 8).