S4 Mathematics Practice Paper: Quadratic Equations and the Discriminant, Quadratic Functions and Graph Transformations, Indices and Logarithmic Equations, Trigonometry (Cosine Rule and Area), Coordinate Geometry of Circles and Straight Lines, Statistics and Probability (100 marks)

Mathematics · S4

S4 Mathematics practice paper: 24 parts, 100 marks, 75 minutes on quadratic equations, function graphs, logarithms, trigonometry, coordinate geometry and probability.

Question 1 - Quadratic Equations and the Discriminant (16 marks)

*For a quadratic equation ax^2 + bx + c = 0 (a not equal 0) the discriminant is D = b^2 - 4ac. If D > 0 there are two distinct real roots, if D = 0 there is a repeated root, if D is negative there is no real root. If the roots are alpha and beta then alpha + beta = -b/a and alpha*beta = c/a.*

#	Question	Answer and marking notes
1	Solve 2x^2 - 7x + 3 = 0 and state the value of the discriminant.	D = (-7)^2 - 4(2)(3) = 49 - 24 = 25. Factorising 2x^2 - 7x + 3 = (2x - 1)(x - 3) gives x = 0.5 or x = 3.
2	If x^2 - 6x + k = 0 has a repeated (double) real root, find k and the repeated root.	A repeated root needs D = 0: (-6)^2 - 4(1)(k) = 36 - 4k = 0, so k = 9. The equation becomes (x - 3)^2 = 0 with repeated root x = 3.
3	For 3x^2 + 5x - 2 = 0, find the discriminant, the nature of the roots, and the sum and product of the roots.	D = 5^2 - 4(3)(-2) = 25 + 24 = 49 > 0, so there are two distinct real roots. Sum of roots = -5/3, product of roots = -2/3.
4	Let alpha and beta be the roots of 2x^2 - 8x + 5 = 0. Find alpha + beta, alpha*beta and alpha^2 + beta^2.	alpha + beta = 8/2 = 4 and alpha*beta = 5/2 = 2.5. Then alpha^2 + beta^2 = (alpha + beta)^2 - 2*alpha*beta = 16 - 5 = 11.

Question 2 - Quadratic Functions and Graph Transformations (16 marks)

The graph of f(x) = 2x^2 - 12x + 13 is a parabola. Translating y = f(x) left by h units and up by k units gives the graph of y = f(x + h) + k.

#	Question	Answer and marking notes
5	Complete the square to write f(x) = 2x^2 - 12x + 13 as a(x - h)^2 + k, and state the vertex and the axis of symmetry.	f(x) = 2(x^2 - 6x) + 13 = 2[(x - 3)^2 - 9] + 13 = 2(x - 3)^2 - 5. Vertex (3, -5), axis of symmetry x = 3.
6	The graph of y = f(x) is translated 2 units left and 4 units up. State the vertex of the new graph and its minimum value.	The new function is y = f(x + 2) + 4 = 2(x - 1)^2 - 9. Vertex (1, -9); since a = 2 > 0 the minimum value is -9.
7	For y = 3 * 2^x, find y when x = 0, 1 and 2, and describe how y changes as x increases by 1.	x = 0 gives y = 3; x = 1 gives y = 6; x = 2 gives y = 12. Each increase of 1 in x multiplies y by 2, i.e. a 100% increase.

#	Question	Answer and marking notes
8	Consider y = -2(x + 1)^2 + 8. State the vertex, the direction of opening, the maximum value and the y-intercept.	Vertex (-1, 8). Since a = -2 is negative so the parabola opens downwards and the maximum value is 8. When x = 0, y = -2(1) + 8 = 6, so the y-intercept is 6.

Question 3 - Indices and Logarithmic Equations (16 marks)

Index laws: a^(m/n) is the n-th root of a^m, and a^(-n) = 1/a^n. Log laws: log(xy) = log x + log y, log(x/y) = log x - log y, log x^n = n log x. After solving a log equation you must check that every argument is positive.

#	Question	Answer and marking notes
9	Without a calculator, evaluate 8^(2/3) * 4^(-1/2).	8^(2/3) = (cube root of 8)^2 = 2^2 = 4 and 4^(-1/2) = 1/sqrt(4) = 0.5, so the product is 4 * 0.5 = 2.
10	Solve 3^(2x - 1) = 27^(x - 2).	Since 27 = 3^3, 27^(x - 2) = 3^(3x - 6). Equating indices: 2x - 1 = 3x - 6, so x = 5 (both sides equal 3^9 = 19,683).
11	Given log 2 = 0.3010 and log 3 = 0.4771 (base 10), find log 12 and log 1.5.	log 12 = 2 log 2 + log 3 = 2(0.3010) + 0.4771 = 1.0791. log 1.5 = log 3 - log 2 = 0.4771 - 0.3010 = 0.1761.
12	Solve log base 2 of x plus log base 2 of (x - 6) equals 4, and explain why one solution must be rejected.	Combining gives log base 2 of x(x - 6) equals 4, so x(x - 6) = 16, i.e. x^2 - 6x - 16 = 0 and (x - 8)(x + 2) = 0, so x = 8 or x = -2. Both arguments must be positive (x > 6), so x = -2 is rejected and x = 8.

Question 4 - Trigonometry: Cosine Rule and Area (18 marks)

In triangle ABC, sides a, b, c are opposite angles A, B, C. Cosine rule: c^2 = a^2 + b^2 - 2ab cos C. Sine rule: a/sin A = b/sin B. Area = (1/2)ab sin C. Give all answers to 3 significant figures.

#	Question	Answer and marking notes
13	In triangle ABC, a = 7 cm, b = 9 cm and C = 52 degrees. Find the length of c.	c^2 = 7^2 + 9^2 - 2(7)(9)cos 52 = 49 + 81 - 77.57 = 52.43, so c = 7.24 cm (3 significant figures).
14	For the same triangle, find the area in cm^2.	Area = 0.5 * 7 * 9 * sin 52 = 31.5 * 0.78801 = 24.8 cm^2 (3 significant figures).
15	In another triangle A = 40 degrees, B = 65 degrees and a = 10 cm. Find b.	By the sine rule, b = 10 * sin 65 / sin 40 = 10 * 0.90631 / 0.64279 = 14.1 cm (3 significant figures).
16	Solve 2 sin^2(theta) - sin(theta) - 1 = 0 for 0 <= theta <= 360 degrees, listing all solutions.	Let s = sin(theta): 2s^2 - s - 1 = 0 gives (2s + 1)(s - 1) = 0, so s = -1/2 or s = 1. sin(theta) = 1 gives 90 degrees; sin(theta) = -1/2 gives 210 and 330 degrees. Hence theta = 90, 210, 330 degrees.

Question 5 - Coordinate Geometry of Circles and Straight Lines (17 marks)

Given A(1, 2) and B(7, 10). Use the distance formula, the midpoint formula and the circle with AB as diameter. The distance from (x0, y0) to the line Ax + By + C = 0 is |Ax0 + By0 + C| / sqrt(A^2 + B^2).

#	Question	Answer and marking notes
17	Find the length of AB and the coordinates of the midpoint of AB.	AB = sqrt(6^2 + 8^2) = sqrt(100) = 10. Midpoint = ((1 + 7)/2, (2 + 10)/2) = (4, 6).
18	Find the equation of the circle with AB as diameter.	Centre is the midpoint (4, 6) and radius = 10/2 = 5, so the equation is (x - 4)^2 + (y - 6)^2 = 25, i.e. x^2 + y^2 - 8x - 12y + 27 = 0.
19	Line L: 3x + 4y - 12 = 0. Find the slope of L, the slope of any line perpendicular to L, and the distance from the origin to L.	L is y = -(3/4)x + 3, so its slope is -3/4 and a perpendicular line has slope 4/3. Distance from the origin = 12/sqrt(9 + 16) = 12/5 = 2.4.
20	Find the equation of the line through (2, -1) parallel to L.	A parallel line has the form 3x + 4y + k = 0. Substituting (2, -1) gives 6 - 4 + k = 0 so k = -2 and the equation is 3x + 4y - 2 = 0.

Question 6 - Statistics and Probability (17 marks)

The data set is 4, 7, 7, 9, 13. Mean = sum / number of values; median is the middle value when ordered; mode is the most frequent value. Standard deviation sigma = sqrt(sum of (x - mean)^2 / n).

#	Question	Answer and marking notes
21	Find the mean, median and mode of the data set.	Sum = 40 and there are 5 values, so mean = 8. The middle value is 7, so median = 7; 7 occurs twice, so mode = 7.
22	If the value 20 is added, find the new mean and say whether the mean or the median is affected more.	New sum = 60 over 6 values, so the new mean is 10. The value 20 is an outlier: the mean moves from 8 to 10 while the median stays 7, so the mean is affected more.
23	The data 2, 4, 6, 8, 10 have mean 6. Find the variance and the standard deviation (3 decimal places).	Sum of squared deviations = 16 + 4 + 0 + 4 + 16 = 40, so variance = 40/5 = 8 and standard deviation = sqrt(8) = 2.828.
24	A bag contains 5 red and 3 blue balls. Two balls are drawn at random without replacement. Find the probability that both are red.	The first ball is red with probability 5/8; then 4 red and 3 blue remain, so the second is red with probability 4/7. The required probability is (5/8)(4/7) = 5/14, about 0.357.