S4 Mathematics Practice Paper: Quadratic Equations and the Discriminant, Quadratic Functions and Graph Transformations, Indices and Logarithmic Equations, Trigonometry (Cosine Rule and Area), Coordinate Geometry of Circles and Straight Lines, Statistics and Probability (100 marks)

Mathematics · S4

S4 Mathematics practice paper: 24 parts, 100 marks, 75 minutes on quadratic equations, function graphs, logarithms, trigonometry, coordinate geometry and probability.

Question 1 - Quadratic Equations and the Discriminant (16 marks)

*For a quadratic equation ax^2 + bx + c = 0 (a not equal 0) the discriminant is D = b^2 - 4ac. If D > 0 there are two distinct real roots, if D = 0 there is a repeated root, if D is negative there is no real root. If the roots are alpha and beta then alpha + beta = -b/a and alpha*beta = c/a.*

#	Question
1	Solve 2x^2 - 7x + 3 = 0 and state the value of the discriminant.
2	If x^2 - 6x + k = 0 has a repeated (double) real root, find k and the repeated root.
3	For 3x^2 + 5x - 2 = 0, find the discriminant, the nature of the roots, and the sum and product of the roots.
4	Let alpha and beta be the roots of 2x^2 - 8x + 5 = 0. Find alpha + beta, alpha*beta and alpha^2 + beta^2.

Question 2 - Quadratic Functions and Graph Transformations (16 marks)

The graph of f(x) = 2x^2 - 12x + 13 is a parabola. Translating y = f(x) left by h units and up by k units gives the graph of y = f(x + h) + k.

#	Question
5	Complete the square to write f(x) = 2x^2 - 12x + 13 as a(x - h)^2 + k, and state the vertex and the axis of symmetry.
6	The graph of y = f(x) is translated 2 units left and 4 units up. State the vertex of the new graph and its minimum value.
7	For y = 3 * 2^x, find y when x = 0, 1 and 2, and describe how y changes as x increases by 1.
8	Consider y = -2(x + 1)^2 + 8. State the vertex, the direction of opening, the maximum value and the y-intercept.

Question 3 - Indices and Logarithmic Equations (16 marks)

Index laws: a^(m/n) is the n-th root of a^m, and a^(-n) = 1/a^n. Log laws: log(xy) = log x + log y, log(x/y) = log x - log y, log x^n = n log x. After solving a log equation you must check that every argument is positive.

#	Question
9	Without a calculator, evaluate 8^(2/3) * 4^(-1/2).

#	Question
10	Solve 3^(2x - 1) = 27^(x - 2).
11	Given log 2 = 0.3010 and log 3 = 0.4771 (base 10), find log 12 and log 1.5.
12	Solve log base 2 of x plus log base 2 of (x - 6) equals 4, and explain why one solution must be rejected.

Question 4 - Trigonometry: Cosine Rule and Area (18 marks)

In triangle ABC, sides a, b, c are opposite angles A, B, C. Cosine rule: c^2 = a^2 + b^2 - 2ab cos C. Sine rule: a/sin A = b/sin B. Area = (1/2)ab sin C. Give all answers to 3 significant figures.

#	Question
13	In triangle ABC, a = 7 cm, b = 9 cm and C = 52 degrees. Find the length of c.
14	For the same triangle, find the area in cm^2.
15	In another triangle A = 40 degrees, B = 65 degrees and a = 10 cm. Find b.
16	Solve 2 sin^2(theta) - sin(theta) - 1 = 0 for 0 <= theta <= 360 degrees, listing all solutions.

Question 5 - Coordinate Geometry of Circles and Straight Lines (17 marks)

Given A(1, 2) and B(7, 10). Use the distance formula, the midpoint formula and the circle with AB as diameter. The distance from (x0, y0) to the line Ax + By + C = 0 is |Ax0 + By0 + C| / sqrt(A^2 + B^2).

#	Question
17	Find the length of AB and the coordinates of the midpoint of AB.
18	Find the equation of the circle with AB as diameter.
19	Line L: 3x + 4y - 12 = 0. Find the slope of L, the slope of any line perpendicular to L, and the distance from the origin to L.
20	Find the equation of the line through (2, -1) parallel to L.

Question 6 - Statistics and Probability (17 marks)

The data set is 4, 7, 7, 9, 13. Mean = sum / number of values; median is the middle value when ordered; mode is the most frequent value. Standard deviation sigma = sqrt(sum of (x - mean)^2 / n).

#	Question
21	Find the mean, median and mode of the data set.
22	If the value 20 is added, find the new mean and say whether the mean or the median is affected more.
23	The data 2, 4, 6, 8, 10 have mean 6. Find the variance and the standard deviation (3 decimal places).
24	A bag contains 5 red and 3 blue balls. Two balls are drawn at random without replacement. Find the probability that both are red.