S4 Mathematics Practice Paper 2 | trigonometry, 3D solids, statistics, linear programming

Senior Secondary Mathematics

S4 Maths practice paper 2 with answers: trigonometric ratios, sine and cosine rules, 3D solids, statistics, coordinate geometry, linear programming and probability.

1. Trigonometric ratios and solving triangles (16 marks)

Show working in all parts; give answers to 3 significant figures or 1 decimal place.

#	Question	Answer and marking notes
1	(a) In a right-angled triangle the opposite side is 7 cm and the adjacent side is 24 cm. Find the hypotenuse and sin θ, cos θ, tan θ to 3 decimal places, where θ is the angle opposite the 7 cm side.	Hypotenuse = sqrt(49 + 576) = 25 cm; sin θ = 7/25 = 0.280, cos θ = 24/25 = 0.960, tan θ = 7/24 = 0.292.
2	(b) From a point on level ground 40 m from the foot of a tower, the angle of elevation of the top is 32 degrees. Find the height of the tower (1 decimal place).	Height = 40 x tan 32 = 24.99 m, about 25.0 m.
3	(c) In triangle ABC, angle A = 48 degrees, angle B = 67 degrees and a = 12 cm. Find angle C and b (1 decimal place).	Angle C = 180 - 48 - 67 = 65 degrees; by the sine rule b = 12 x sin 67 / sin 48 = 14.9 cm.
4	(d) Two sides of a triangle are 9 cm and 12 cm with an included angle of 55 degrees. Find the third side and the area (1 decimal place).	Third side = sqrt(81 + 144 - 216 cos 55) = 10.1 cm; area = 0.5 x 9 x 12 x sin 55 = 44.2 cm2.

2. 3D solids and measurement (18 marks)

Take pi as 3.142 and give answers to 1 decimal place.

#	Question	Answer and marking notes
5	(a) A cone has base radius 5 cm and height 12 cm. Find its slant height, volume and total surface area.	Slant height = 13 cm; volume = (1/3) x 3.142 x 25 x 12 = 314.2 cm3; total surface area = 3.142 x 5 x 13 + 3.142 x 25 = 282.8 cm2.
6	(b) A sphere has radius 5 cm. Find its surface area and volume.	Surface area = 4 x 3.142 x 25 = 314.2 cm2; volume = (4/3) x 3.142 x 125 = 523.7 cm3.
7	(c) A solid cylinder has radius 4 cm and height 10 cm. Find its total surface area and volume.	Total surface area = 2 x 3.142 x 16 + 2 x 3.142 x 4 x 10 = 351.9 cm2; volume = 3.142 x 16 x 10 = 502.7 cm3.
8	(d) A box measures 30 cm by 20 cm by 15 cm. Find its surface area. How many 5 cm cubes fill it? If 4 cm cubes are packed in rows instead, what is the maximum number and what volume of the box is left empty?	Surface area = 2 x (600 + 300 + 450) = 2700 cm2; 5 cm cubes: 6 x 4 x 3 = 72; 4 cm cubes: 7 x 5 x 3 = 105 using 6720 cm3, leaving 9000 - 6720 = 2280 cm3 empty.

3. Statistics and data analysis (16 marks)

Test scores of 8 students in ascending order: 12, 15, 15, 18, 20, 22, 25, 28. Show working or explain.

#	Question	Answer and marking notes
9	(a) Find the mean (2 decimal places) and the range.	Mean = 155 / 8 = 19.38; range = 28 - 12 = 16.
10	(b) Find the median and the mode.	With 8 values the median = (18 + 20) / 2 = 19; mode = 15, which appears twice.
11	(c) Find the lower quartile, upper quartile and interquartile range.	Lower half 12, 15, 15, 18 gives Q1 = 15.0; upper half 20, 22, 25, 28 gives Q3 = 23.5; IQR = Q3 - Q1 = 8.5.
12	(d) Find the standard deviation of the 8 scores (population, 2 decimal places).	Sum of squared deviations = 207.875; variance = 25.984; standard deviation = 5.10.

4. Coordinate geometry and the circle (18 marks)

Show working throughout.

#	Question	Answer and marking notes
13	(a) Given A(2, -1) and B(7, 11), find the length of AB and the coordinates of its midpoint.	AB = sqrt(25 + 144) = 13; midpoint = (4.5, 5).
14	(b) Find the slope of the line through A(2, -1) and B(7, 11) and write its equation in the form ax + by + c = 0.	Slope = 12/5 = 2.4; y + 1 = 2.4(x - 2), which simplifies to 12x - 5y - 29 = 0.
15	(c) Find the coordinates of the intersection of the lines 3x + y = 10 and x - 2y = 1.	Substituting y = 10 - 3x into x - 2y = 1 gives 7x = 21, so x = 3 and y = 1; the lines meet at (3, 1).
16	(d) A circle has equation x^2 + y^2 - 6x + 8y - 11 = 0. Find its centre, radius, circumference and area (pi = 3.142, 1 decimal place).	(x - 3)^2 + (y + 4)^2 = 36; centre (3, -4), radius 6; circumference = 37.7; area = 113.1.

5. Linear inequalities and linear programming (16 marks)

Show working or explain throughout.

#	Question	Answer and marking notes
17	(a) Solve the inequality 5x - 7 > 2x + 8.	5x - 2x > 8 + 7 gives 3x > 15, so x > 5.
18	(b) Solve the inequality -3x + 4 >= 19.	-3x >= 15; dividing by a negative reverses the sign, so x <= -5.
19	(c) Subject to x + y = 0, y >= 0, find the maximum of P = 3x + 2y and the point where it occurs.	Vertices (0,0), (30,0), (20,20), (0,40) give P = 0, 90, 100, 80; the maximum 100 occurs at (20, 20).
20	(d) Using the region in (c), find its area in square units.	By the shoelace formula for (0,0), (30,0), (20,20), (0,40): the sum is 1400, so the area is 700 square units.

6. Probability and counting (16 marks)

Show working throughout; probabilities may be given as fractions or decimals.

#	Question	Answer and marking notes
21	(a) A bag holds 5 red, 3 blue and 2 green balls. One ball is drawn at random. Find P(red) and P(not green).	There are 10 balls; P(red) = 5/10 = 0.5 and P(not green) = 8/10 = 0.8.
22	(b) Two balls are drawn from the same bag without replacement. Find the probability that both are red, as a fraction.	P = 5/10 x 4/9 = 2/9, about 0.222.
23	(c) Five students stand in a row. Find the number of arrangements; if two named students must stand together, find the number of arrangements.	Total = 5! = 120; treating the pair as one unit gives 2 x 4! = 48.
24	(d) Three students are chosen from 8 to form a committee. Find the number of committees and the probability that one named student is included.	Number of committees = C(8,3) = 56; the probability that the named student is included is 3/8 = 0.375.