S4 Mathematics (Compulsory Part) Practice Paper: Quadratic Equations, Indices and Logarithms, Sequences, Circle Properties, Probability and Statistics

Secondary 4 | Mathematics (Compulsory Part) | Time: 75 minutes | Total: 100 marks | 24 questions

Part 1: Quadratic Equations and the Discriminant (16 marks)

Consider the quadratic equation x^2 - 6x + 5 = 0 and the function y = x^2 - 6x + 5.

1. Solve the equation x^2 - 6x + 5 = 0.

Answer: Factorising gives (x - 1)(x - 5) = 0, so x = 1 or x = 5.

2. Find the discriminant of 2x^2 + 3x + 4 = 0 and state the number of real roots.

Answer: Discriminant = 3^2 - 4 x 2 x 4 = 9 - 32 = -23. Since it is negative, there are no real roots.

3. If the equation x^2 + kx + 9 = 0 has a repeated root, find the two possible values of k.

Answer: A repeated root means the discriminant is zero: k^2 - 4 x 1 x 9 = 0, so k^2 = 36 and k = 6 or k = -6.

4. Find the axis of symmetry and the coordinates of the vertex of y = x^2 - 6x + 5.

Answer: Axis of symmetry x = 3. Substituting gives y = 9 - 18 + 5 = -4, so the vertex is (3, -4).

Part 2: Indices and Logarithms (16 marks)

Simplify each of the following. Exact values are required unless stated otherwise. Given log10(2) = 0.3010.

5. Find the value of log2(32).

Answer: Since 32 = 2^5, log2(32) = 5.

6. Using log10(2) = 0.3010, find log10(2000) correct to 4 decimal places.

Answer: 2000 = 2 x 10^3, so log10(2000) = 0.3010 + 3 = 3.3010.

7. Solve the equation 3^(2x) = 27.

Answer: Since 27 = 3^3, 2x = 3 and x = 1.5.

8. Find the value of (log3(9))(log2(8)).

Answer: log3(9) = 2 and log2(8) = 3, so the product is 2 x 3 = 6.

Part 3: Arithmetic and Geometric Sequences (18 marks)

An arithmetic sequence has first term 7 and common difference 4. A geometric sequence has first term 3 and common ratio 2.

9. Find the 20th term of the arithmetic sequence.

Answer: 20th term = 7 + (20 - 1) x 4 = 7 + 76 = 83.

10. Find the sum of the first 20 terms of the arithmetic sequence.

Answer: Sum = 20 x (7 + 83) / 2 = 20 x 90 / 2 = 900.

11. Find the 8th term of the geometric sequence.

Answer: 8th term = 3 x 2^7 = 3 x 128 = 384.

12. Find the sum of the first 8 terms of the geometric sequence.

Answer: Sum = 3 x (2^8 - 1) / (2 - 1) = 3 x 255 = 765.

Part 4: Circle Properties and Sector Area (18 marks)

Circle C has centre (2, -3) and radius 5. Take pi = 3.1416 where needed.

13. Write down the equation of circle C.

Answer: The equation is (x - 2)^2 + (y + 3)^2 = 5^2 = 25.

14. A chord is at a distance of 5 from the centre of a circle of radius 13. Find the length of the chord.

Answer: Half the chord = sqrt(13^2 - 5^2) = sqrt(144) = 12, so the chord is 2 x 12 = 24.

15. From an external point P, a tangent PT is drawn to a circle of radius 6. If PO = 10, find PT.

Answer: Since OT is perpendicular to PT, PT^2 = 10^2 - 6^2 = 64, so PT = 8.

16. A sector has radius 9 and central angle 80 degrees. Find its area correct to 1 decimal place.

Answer: Area = (80 / 360) x pi x 9^2 = 18 pi = 18 x 3.1416 = 56.5 square units.

Part 5: Permutations, Combinations and Probability (16 marks)

Show your working in each part.

17. In how many ways can 5 students be arranged in a row?

Answer: Number of arrangements = 5! = 120.

18. In how many ways can a group of 3 students be chosen from 8 students?

Answer: Number of combinations = C(8, 3) = 56.

19. A bag contains 4 red balls and 6 blue balls. One ball is drawn at random. Find the probability that it is red.

Answer: There are 10 balls in total, so P(red) = 4 / 10 = 0.4.

20. Two fair dice are thrown together. Find the probability that the sum of the scores is 7.

Answer: There are 36 equally likely outcomes and 6 give a sum of 7, so the probability is 6 / 36 = 1/6.

Part 6: Statistics (16 marks)

Data set A: 3, 7, 7, 9, 14. Data set B: 2, 4, 6, 8, 10.

21. Find the mean of data set A.

Answer: Mean = (3 + 7 + 7 + 9 + 14) / 5 = 40 / 5 = 8.

22. Find the median of data set A.

Answer: Arranged in order: 3, 7, 7, 9, 14. With 5 values the median is the 3rd value, 7.

23. Find the mode of data set A.

Answer: The value 7 occurs twice, more often than any other, so the mode is 7.

24. Find the standard deviation of data set B correct to 3 significant figures.

Answer: Mean = 6; sum of squared deviations = 16 + 4 + 0 + 4 + 16 = 40; variance = 40 / 5 = 8; standard deviation = sqrt(8) = 2.83.

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