S5 Mathematics (Extended Part Module 1) Practice Paper | Binomial theorem, exponential and logarithmic functions, differentiation, integration, probability distributions

S5 Mathematics (Extended Part Module 1) | Time allowed: 75 minutes | Total: 100 marks | 24 questions

Question 1 Binomial Theorem

Answer the following questions and show your working.

1. Find the value of C(6, 3). (2 marks)

2. Find the coefficient of x^3 in the expansion of (1 + 2x)^6. (4 marks)

3. Find the coefficient of x^2 in the expansion of (2 - x)^5. (4 marks)

4. Find the constant term in the expansion of (x + 2/x)^4. (6 marks)

Question 2 Exponential and Logarithmic Functions

Solve the following equations and state the formula used in the applied part.

5. Solve 3^(2x) = 81. (3 marks)

6. Solve log2 x + log2 (x - 2) = 3. (5 marks)

7. Simplify (log 27) / (log 3). (3 marks)

8. A machine worth $50 000 depreciates by 15% each year. Find the least number of years for its value to fall below $20 000. (5 marks)

Question 3 Differentiation and Tangents

Differentiate the following functions and answer the tangent question.

9. Let y = (3x + 1)^5. Find dy/dx. (3 marks)

10. Let y = x^2 ln x. Find dy/dx. (4 marks)

11. Let y = (2x - 1)/(x + 3). Find dy/dx. (5 marks)

12. A curve is y = x^3 - 3x. Find the equation of the tangent at x = 2. (6 marks)

Question 4 Applications of Differentiation

Find stationary points, extrema and a related rate.

13. A curve is y = x^3 - 6x^2 + 9x. Find all stationary points and determine their nature. (5 marks)

14. A rectangular enclosure is made with 40 m of fencing. Find the largest possible area. (5 marks)

15. The volume of a sphere is V = (4/3)pi r^3. When r = 5 cm and r increases at 0.2 cm/s, find dV/dt. (3 marks)

16. For the curve y = x^3 - 6x^2 + 9x, find the range of x where the curve is concave upward. (3 marks)

Question 5 Integration and Area

Evaluate the following integrals and the area under a curve.

17. Find the integral of (6x^2 - 4x + 3) with respect to x. (3 marks)

18. Evaluate the definite integral of (x^3 + 1) from x = 1 to x = 2. (4 marks)

19. Find the area bounded by the curve y = x(4 - x) and the x-axis for 0 <= x <= 4. (5 marks)

20. Find the integral of (2x + 1)^3 with respect to x. (4 marks)

Question 6 Binomial and Poisson Distributions

Let X be a binomial random variable and Y a Poisson random variable.

21. Let X be binomial with n = 10 and p = 0.3. Find P(X = 3) correct to 3 significant figures. (5 marks)

22. Using the same X, find P(X >= 1) correct to 3 significant figures. (4 marks)

23. Find the mean and the variance of X. (4 marks)

24. Let Y be Poisson with mean 4. Find P(Y = 0) and P(Y <= 1) correct to 3 significant figures. (5 marks)
