S5 Mathematics Extended Part Module 1 Practice Paper: Binomial Theorem, Exponentials and Logarithms, Limits and Derivatives, Applications of Differentiation, Integration, Probability Distributions

S5 | Mathematics Extended Part Module 1 | Time: 75 minutes | Total: 100 marks | 24 questions

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Part A Binomial Theorem (16 marks)

Answer all questions and show your working.

1. Find the coefficient of x^3 in the expansion of (1 + 2x)^7 and write down that term. [4]

Answer: Coefficient = C(7,3) x 2^3 = 35 x 8 = 280, so the term is 280x^3.

2. Find the constant term in the expansion of (x + 2/x^2)^6. [4]

Answer: General term = C(6,r) x^(6-r) (2x^-2)^r = C(6,r) 2^r x^(6-3r). Setting 6 - 3r = 0 gives r = 2, so the constant term is C(6,2) x 2^2 = 15 x 4 = 60.

3. Find the sum of all coefficients in the expansion of (3 - x)^8 and state which value of x is substituted. [4]

Answer: The sum of coefficients is obtained by putting x = 1: (3 - 1)^8 = 2^8 = 256.

4. How many terms are there in the expansion of (2 + x)^12? Find the coefficient of the middle term. [4]

Answer: There are 13 terms; the middle one is the 7th term with coefficient C(12,6) = 924.

Part B Exponentials and Logarithms (16 marks)

Show your working; give numerical answers to the stated accuracy.

5. Find the value of log base 9 of 27, giving your answer as a fraction. [4]

Answer: log_9 27 = log_3 27 / log_3 9 = 3 / 2 = 1.5.

6. Solve 2^x = 5, giving your answer correct to 4 decimal places. [4]

Answer: x = log_2 5 = ln 5 / ln 2 = 2.3219.

7. A radioactive substance has a half-life of 8 years and an initial mass of 640 g. (a) Find the mass after 24 years. (b) Find the time for the mass to fall to 40 g. [4]

Answer: (a) 640 x (1/2)^(24/8) = 640 / 8 = 80 g. (b) 640 / 40 = 16 = 2^4, so 4 x 8 = 32 years.

8. A solution has hydrogen ion concentration 2.5 x 10^-5 mol/L. Find its pH correct to 1 decimal place. [4]

Answer: pH = -log(2.5 x 10^-5) = 5 - log 2.5 = 5 - 0.398 = 4.6.

Part C Limits and Derivatives (16 marks)

State the rules used; answers may be exact or to the stated accuracy.

9. Evaluate lim (x -> 3) of (x^2 - 9) / (x - 3). [4]

Answer: Factorise the numerator as (x - 3)(x + 3), cancel (x - 3) and the limit is x + 3 = 6.

10. Let y = x^2 e^(3x). Find dy/dx at x = 1, exact and to 3 decimal places. [4]

Answer: By the product rule dy/dx = e^(3x)(2x + 3x^2); at x = 1 this is 5e^3, about 100.428.

11. Let y = ln(3x^2 + 1). Find dy/dx at x = 2 as a fraction. [4]

Answer: dy/dx = 6x / (3x^2 + 1); at x = 2 this is 12/13, about 0.923.

12. Let y = sqrt(2x + 5). Use the chain rule to find dy/dx at x = 2. [4]

Answer: y = (2x + 5)^(1/2) so dy/dx = 1 / sqrt(2x + 5); at x = 2 this is 1/3, about 0.333.

Part D Applications of Differentiation (16 marks)

State the conditions used and justify the nature of each stationary point.

13. The curve is y = x^3 - 2x. Find the equation of the tangent at x = 1. [4]

Answer: y' = 3x^2 - 2, so the slope at x = 1 is 3 - 2 = 1. The point is (1, -1) and the tangent is y = x - 2.

14. The curve is y = x^3 - 3x^2 + 4. Find all stationary points and determine whether each is a maximum or a minimum. [4]

Answer: y' = 3x^2 - 6x = 3x(x - 2), so x = 0 and x = 2. With y'' = 6x - 6, at x = 0 y'' = -6 < 0 giving a maximum (0, 4), and at x = 2 y'' = 6 > 0 giving a minimum (2, 0).

15. A rectangle has a fixed perimeter of 40 cm. Find its maximum area and the corresponding side lengths. [4]

Answer: If one side is a the other is 20 - a, so A = a(20 - a) and A' = 20 - 2a = 0 gives a = 10. The maximum area is 100 cm^2, a 10 x 10 square.

16. The radius of a sphere increases at 0.5 cm/s. Find the rate of increase of volume when the radius is 3 cm, exact and to 3 decimal places. [4]

Answer: V = (4/3)pi r^3 so dV/dt = 4 pi r^2 (dr/dt) = 4 pi (9)(0.5) = 18 pi, about 56.549 cm^3/s.

Part E Integration and Area (16 marks)

Show the primitive or formula used; exact answers are acceptable.

17. Evaluate the integral from 0 to 2 of (3x^2 + 2x) dx. [4]

Answer: The primitive is x^3 + x^2; evaluating from 0 to 2 gives (8 + 4) - 0 = 12.

18. Find the area enclosed by y = x and y = x^2 for 0 <= x <= 1. [4]

Answer: On 0 <= x <= 1, x >= x^2, so the area is the integral of (x - x^2) = 1/2 - 1/3 = 1/6, about 0.167.

19. Evaluate the integral from 1 to e of (1/x) dx. [4]

Answer: The primitive is ln x, so the value is ln e - ln 1 = 1 - 0 = 1.

20. The curve y = x^2 from x = 0 to x = 1 is rotated about the x-axis. Find the volume, in terms of pi and to 3 decimal places. [4]

Answer: V = pi times the integral of x^4 from 0 to 1 = pi/5, about 0.628.

Part F Probability and Distributions (20 marks)

State the distribution and formula used and show the substitution.

21. X ~ B(10, 0.3). Find P(X = 3) correct to 4 decimal places. [5]

Answer: P(X = 3) = C(10,3)(0.3)^3(0.7)^7 = 120 x 0.027 x 0.0823543, about 0.2668.

22. X ~ B(20, 0.3). Find the mean, variance and standard deviation (3 decimal places). [5]

Answer: E(X) = np = 6; Var(X) = np(1 - p) = 20 x 0.3 x 0.7 = 4.2; standard deviation = sqrt(4.2), about 2.049.

23. X ~ N(50, 8^2). Find P(X > 62) correct to 4 decimal places. [5]

Answer: z = (62 - 50) / 8 = 1.5, so P(X > 62) = P(Z > 1.5) = 1 - 0.9332 = 0.0668.

24. X ~ N(50, 8^2). Find P(42 < X < 58) to 4 decimal places and relate it to the 68% rule. [5]

Answer: The z-scores are -1 and 1, so P = 0.6827, about 68%, matching the empirical rule for mu +/- 1 sigma.
