S5 Mathematics Extended Part Module 1 Practice Paper: Binomial Theorem, Exponential and Logarithmic Functions, Differentiation and Extrema, Integration and Area, Binomial Distribution, Growth and Optimisation

S5 Mathematics Extended Part Module 1 | Time: 75 minutes |

Section A: Binomial Theorem (16 marks)

Show all necessary working and simplify your answers.

1. Find the coefficient of x^3 in the expansion of (1 + 2x)^10.

The x^3 term is C(10, 3)(2x)^3, so the coefficient is 120 x 8 = 960.

2. Find the constant term in the expansion of (x – 3/x)^6.

General term = C(6, r)(–3)^r x^(6–2r). Setting 6 – 2r = 0 gives r = 3, so the constant term is C(6, 3)(–3)^3 = 20 x (-27) = -540.

3. Find the sum of all coefficients in the expansion of (2 + x)^7.

Put x = 1: (2 + 1)^7 = 3^7 = 2187.

4. Find the coefficient of x^5 in the expansion of (1 – x)^8.

Coefficient = C(8, 5)(–1)^5 = 56 x (–1) = -56.

Section B: Exponential and Logarithmic Functions (16 marks)

Unless otherwise stated, give answers correct to 3 significant figures.

5. Solve 4^(x+1) = 8^(2x–1).

Rewrite as 2^(2x+2) = 2^(6x–3), so 2x + 2 = 6x – 3 and x = 5/4 = 1.25.

6. Solve $\log_2 x + \log_2 (x - 3) = 2$.

Combine: $\log_2[x(x - 3)] = 2$, so $x^2 - 3x - 4 = 0$, $(x - 4)(x + 1) = 0$. Rejecting $x = -1$ gives $x = 4$.

7. A bacteria culture grows as N = 500 e^(kt). After 3 hours N = 4000. (a) Find k. (b) How long does it take to reach 20000?

(a) 4000 = 500 e^(3k) gives e^(3k) = 8, so k = ln 8 / 3 ≈ 0.693. (b) 20000 = 500 e^(kt) gives e^(kt) = 40, so t = ln 40 / k ≈ 5.32 hours.

8. A radioactive substance has a half-life of 12 years. (a) Find the decay constant k. (b) What percentage remains after 30 years?

(a) e^(–12k) = 1/2 gives k = ln 2 / 12 ≈ 0.0578 per year. (b) Remaining fraction = e^(–k × 30) ≈ 0.177, i.e. about 17.7%.

Section C: Differentiation and Applications (20 marks)

Use standard differentiation rules and state the rule used.

9. Let $y = (3x^2 + 1)(x - 2)$. (a) Find dy/dx. (b) Find dy/dx when $x = 2$.

Expanding, $y = 3x^3 - 6x^2 + x - 2$, so $dy/dx = 9x^2 - 12x + 1$. At $x = 2$, $dy/dx = 36 - 24 + 1 = 13$.

10. Let $y = (2x + 5)/(x - 1)$. Use the quotient rule to find dy/dx in simplest form.

$dy/dx = [2(x - 1) - (2x + 5)] / (x - 1)^2 = (-7) / (x - 1)^2$.

11. Let $y = (3x^2 + 4)^5$. Use the chain rule to find dy/dx.

$dy/du = 5u^4$ and $du/dx = 6x$, so $dy/dx = 30x(3x^2 + 4)^4$.

12. Let $f(x) = x^3 - 6x^2 + 9x + 2$. Find the stationary points, determine whether each is a maximum or a minimum, and give the corresponding values of $f(x)$.

$f'(x) = 3(x - 1)(x - 3)$, so stationary points at $x = 1$ and $x = 3$. Since $f''(1) = -6$ is negative the point $(1, 6)$ is a maximum, and since $f''(3) = 6$ is positive the point $(3, 2)$ is a minimum.

Section D: Integration and Area (20 marks)

Include the constant of integration; list the ordinates used in any approximation.

13. Find $\int (6x^2 - 4x + 3)\,dx$.

The integral is $2x^3 - 2x^2 + 3x + C$.

14. Evaluate $\int_0^2 (3x^2 + 2x)\,dx$.

An antiderivative is $x^3 + x^2$, so the value is $(8 + 4) - 0 = 12$.

15. Find the area enclosed by the curve $y = 4 - x^2$ and the x-axis, correct to 3 significant figures.

The curve meets the x-axis at $x = \pm 2$, so the area is $2 \times (8 - 8/3) = 32/3 \approx 10.67$ square units.

16. Use the trapezoidal rule with 4 equal subintervals to approximate $\int_0^2 x^3\,dx$, then compare with the exact value and state the absolute error.

With $h = 0.5$ the ordinates are 0, 0.125, 1, 3.375 and 8. The estimate is $(0.5/2)[0 + 8 + 2(0.125 + 1 + 3.375)] = 4.25$, the exact value is 4, so the absolute error is 0.25.

Section E: Binomial Distribution and Expectation (16 marks)

Give probabilities correct to 4 decimal places.

17. Let $X \sim B(10, 0.3)$. Find $P(X = 3)$.

$P(X = 3) = C(10, 3)(0.3)^3(0.7)^7 \approx 0.2668$.

18. For the same distribution, find the mean and the variance of X.

Mean $= np = 10 \times 0.3 = 3.0$; variance $= np(1 - p) = 2.0999999999999996$.

19. For the same distribution, find P(X ≥ 1).

P(X ≥ 1) = 1 − (0.7)^10 = 1 − 0.0282475 ≈ 0.9718.

20. A player scores with probability 0.8 on each free throw and takes 5 throws, so Y ~ B(5, 0.8). (a) Find the probability of scoring all 5. (b) Find P(Y ≥ 4).

(a) P(Y = 5) = 0.8^5 = 0.32768. (b) P(Y ≥ 4) = 0.3277 + 0.32768 = 0.73728.

Section F: Applications and Optimisation (12 marks)

State the formula or test you use.

21. An investment grows continuously as A = 10000 e^(0.05t), where t is in years. Find the amount when t = 10, correct to the nearest dollar.

A = 10000 x e^(0.5) ≈ 16487.21, i.e. about 16487 dollars.

22. The production cost in hundreds of dollars is C(x) = 0.5x² − 20x + 500 for output x. Find the output that minimises cost and the minimum cost.

C'(x) = x − 20, so x = 20; since C''(x) = 1 > 0 this gives a minimum. The minimum cost is 300 (hundreds of dollars).

23. Find the average value of f(x) = x² on the interval [0, 3].

Average value = (1/3) $\int_0^3 x^2\,dx$ = (1/3)(9) = 3.

24. A sphere has volume V = (4/3)πr³. (a) Find dV/dr when r = 5. (b) If the radius increases at 0.02 cm/s, find the rate of change of volume then, correct to 3 significant figures.

(a) dV/dr = 4πr² = 100π ≈ 314.16 cm² at r = 5. (b) dV/dt = 100π x 0.02 = 2π ≈ 6.28 cm³/s.

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