S5 Mathematics Extended Part Module 2 Practice Paper: Matrices and Determinants, Vectors, Binomial Theorem and Mathematical Induction, Calculus of Exponential and Logarithmic Functions, Integration Techniques, Trigonometry and Applications (100 marks)

S5 | Mathematics Extended Part Module 2 | Time: 75 minutes | Total: 100 marks | 24 questions

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Part 1: Matrices and Determinants (15 marks)

Show all working.

1. Let A = [[2, 3], [1, 4]] and B = [[1, -1], [2, 0]]. Find AB. [4]

2. Let N = [[k, 2], [3, k + 1]]. Find all values of k for which N is singular (not invertible). [4]

3. Let M = [[4, 1], [2, 3]]. Find det M and the inverse matrix M^(-1). [4]

4. Solve the system 3x + y = 5 and 2x + 4y = 10 by a matrix method. [3]

Part 2: Vectors (17 marks)

Give angles to 0.1 degree; keep surd form otherwise.

5. Let a = (3, -1, 2) and b = (1, 4, -2). Find a.b and the angle between a and b (to 0.1 degree). [4]

6. Given a = (2, t, 1) and b = (1, -1, 3) are perpendicular, find t. [4]

7. Let a = (2, -1, 3) and b = (1, 4, -2). Find a x b. [5]

8. The position vectors of A and B are a = (1, 2, 3) and b = (2, -1, 1). Find the area of triangle OAB. [4]

Part 3: Binomial Theorem and Mathematical Induction (17 marks)

Induction needs the base case, assumption and inductive step.

9. Find the coefficient of x^3 in the expansion of (2 + x)^7. [4]

10. Find the term independent of x in the expansion of (x^2 + 2/x)^9. [4]

11. Prove by mathematical induction that 1 + 5 + 9 + ... + (4n - 3) = n(2n - 1) for all positive integers n. [4]

12. Given that the sum of k(k+1) from k = 1 to n equals n(n+1)(n+2)/3, find the sum from k = 1 to 20. [5]

Part 4: Exponential and Logarithmic Functions, Differentiation (17 marks)

State the rule used.

13. Let y = x^2 e^(3x). Find dy/dx. [4]

14. Let y = ln(x^2 + 1). Find dy/dx. [4]

15. Let y = x ln x for x > 0. Find the equation of the tangent to the curve at x = e. [5]

16. Solve e^(2x) - 3e^x + 2 = 0. [4]

Part 5: Integration (17 marks)

Show the limits substituted for definite integrals.

17. Find the indefinite integral of x e^x with respect to x. [4]

18. Evaluate the definite integral of ln x from x = 1 to x = e. [4]

19. Evaluate the definite integral of 4x/(x^2 + 1) from x = 0 to x = 2. [5]

20. Find the area of the region bounded by y = x and y = x^2 between x = 0 and x = 1. [4]

Part 6: Trigonometry and Applications (17 marks)

List every solution in the range 0 <= x < 2 pi.

21. Prove that (sin theta + cos theta)^2 = 1 + sin 2 theta. [4]

22. Solve 2 sin^2 x - sin x - 1 = 0 for 0 <= x < 2 pi. [5]

23. Let y = x^3 - 3x^2 + 4. Find the local maximum and local minimum points of the curve. [4]

24. The radius r of a circle increases at 2 cm per second. Find the rate of increase of the area A when r = 5 cm. [4]
