S5 Mathematics Extended Part Module 1 Practice Paper 1 | Differentiation, integration, binomial, Poisson, normal and estimation

Senior Secondary Mathematics Extended Part Module 1

S5 M1 practice paper with worked solutions: differentiation, integration and areas, binomial and Poisson, normal distribution, confidence interval and hypothesis testing.

1. Differentiation of exponential and logarithmic functions (18 marks)

Give answers to 4 decimal places and show the differentiation steps.

#	Question	Answer and marking notes
1	(a) Let y = 3x^2 e^(2x). Find dy/dx and evaluate it at x = 1.	dy/dx = 6x e^(2x) + 6x^2 e^(2x) = 6x(1 + x) e^(2x); at x = 1, 6 x 2 x e^2 = 12e^2 = 88.6687.
2	(b) Let y = ln(4x + 1). Find dy/dx and evaluate it at x = 2.	dy/dx = 4/(4x + 1); at x = 2, 4/9 = 0.4444.
3	(c) Let y = e^(3x)/x. Find dy/dx and evaluate it at x = 1.	dy/dx = [3x e^(3x) - e^(3x)]/x^2 = e^(3x)(3x - 1)/x^2; at x = 1, 2e^3 = 40.1711.
4	(d) Let y = x^2 ln x. Find dy/dx and evaluate it at x = e.	dy/dx = 2x ln x + x^2(1/x) = 2x ln x + x; at x = e, 2e + e = 3e = 8.1548.

2. Applications of differentiation: stationary points, tangents and related rates (16 marks)

Show f'(x) and justify each conclusion.

#	Question	Answer and marking notes
5	(a) Let f(x) = x^3 - 6x^2 + 9x + 2. Find f'(x) and the coordinates of all stationary points.	f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3); the stationary points are (1, 6) and (3, 2).
6	(b) Find the equation of the tangent to y = x^2 + 3x at x = 2.	Slope = 2(2) + 3 = 7; the point is (2, 10); tangent: y = 7x - 4.
7	(c) Let g(x) = x^3 - 3x. Use the second derivative test to classify the stationary points at x = 1 and x = -1.	g'(x) = 3x^2 - 3, so x = +/-1; g”(x) = 6x, g”(1) = 6 > 0 (minimum), g”(-1) = -6 < 0 (maximum).
8	(d) The radius of a sphere increases at 0.5 cm per second. Find the rate of increase of its volume when the radius is 3 cm.	V = (4/3)pi r^3, dV/dt = 4 pi r^2 (dr/dt) = 4 pi (9)(0.5) = 18 pi = 56.55 cm^3/s.

3. Integration and area between curves (16 marks)

Give definite integral answers to 4 decimal places and show the integration steps.

#	Question	Answer and marking notes
9	(a) Find the indefinite integral of 6x^2 + 4x - 3 with respect to x.	The integral is 2x^3 + 2x^2 - 3x + C.

#	Question	Answer and marking notes
10	(b) Evaluate the definite integral of 3x^2 from x = 1 to x = 2.	Integral = [x^3] from 1 to 2 = 8 - 1 = 7.
11	(c) Evaluate the integral of x e^(x^2) from x = 0 to x = 1.	Let u = x^2, du = 2x dx: integral = (1/2)[e^(x^2)] from 0 to 1 = (e - 1)/2 = 0.8591.
12	(d) Find the area enclosed by y = x^2 and y = x for 0 <= x <= 1.	Area = integral from 0 to 1 of (x - x^2) dx = 1/2 - 1/3 = 1/6 = 0.1667 square units.

4. Binomial and Poisson distributions (18 marks)

Give probabilities to 4 decimal places and state the formula used.

#	Question	Answer and marking notes
13	(a) X ~ B(10, 0.3). Find P(X = 3).	P = C(10,3)(0.3)^3(0.7)^7 = 120 x 0.027 x 0.0824 = 0.2668.
14	(b) Using (a), find the mean, variance and standard deviation of X.	Mean = np = 3; variance = np(1 - p) = 2.1; standard deviation = sqrt(2.1) = 1.4491.
15	(c) Y ~ Po(4). Find P(Y = 0) and P(Y >= 1).	P(Y = 0) = e^-4 = 0.0183; P(Y >= 1) = 1 - e^-4 = 0.9817.
16	(d) Cars pass a point on a road at an average of 2.5 per 5 minutes. Using a Poisson model, find the probability that exactly 2 cars pass in 5 minutes.	lambda = 2.5; P(Y = 2) = e^-2.5 (2.5^2)/2! = 0.2565.

5. The normal distribution (16 marks)

Standard normal tables may be used: Phi(1) = 0.8413, Phi(1.5) = 0.9332, Phi(-1) = 0.1587, z(0.9) = 1.2816.

#	Question	Answer and marking notes
17	(a) X ~ N(60, 8^2). Find P(X < 68).	z = (68 - 60)/8 = 1; P = Phi(1) = 0.8413.
18	(b) Using (a), find P(52 < X < 72).	z = -1 and 1.5; P = Phi(1.5) - Phi(-1) = 0.9332 - 0.1587 = 0.7745.
19	(c) Using (a), find k such that P(X < k) = 0.9.	z = 1.2816; k = 60 + 1.2816 x 8 = 70.2528.
20	(d) Using (a), a sample of 16 is taken. Find P(sample mean > 63).	Standard error = 8/sqrt(16) = 2; z = (63 - 60)/2 = 1.5; P = 1 - Phi(1.5) = 0.0668.

6. Point estimation, confidence interval and hypothesis testing (16 marks)

Give answers to 4 decimal places and state the test conclusion.

#	Question	Answer and marking notes
21	(a) A sample is 12, 15, 9, 14, 10. Find the sample mean.	Mean = (12 + 15 + 9 + 14 + 10)/5 = 60/5 = 12.
22	(b) Using (a), find the unbiased sample variance.	Sum of squared deviations = 0 + 9 + 9 + 4 + 4 = 26; unbiased variance = 26/(5 - 1) = 6.5000.
23	(c) A sample of 25 has mean 12 and standard deviation 5. Find the 95% confidence interval for the population mean.	Standard error = 5/sqrt(25) = 1; interval = 12 +/- 1.96 x 1 = (10.0400, 13.9600).
24	(d) Test H0: mu = 50 against H1: mu != 50 with a sample of 36, mean 52.4 and standard deviation 9. Find the test statistic and conclude at the 5% level.	z = (52.4 - 50)/(9/sqrt(36)) = 2.4/1.5 = 1.6; since 1.6 < 1.96, do not reject H0: there is insufficient evidence that the population mean differs from 50.