S5 Mathematics Extended Part Module 1 Practice Paper 1 | Differentiation, integration, binomial, Poisson, normal and estimation

Senior Secondary Mathematics Extended Part Module 1

S5 M1 practice paper with worked solutions: differentiation, integration and areas, binomial and Poisson, normal distribution, confidence interval and hypothesis testing.

1. Differentiation of exponential and logarithmic functions (18 marks)

Give answers to 4 decimal places and show the differentiation steps.

#	Question
1	(a) Let y = 3x^2 e^(2x). Find dy/dx and evaluate it at x = 1.
2	(b) Let y = ln(4x + 1). Find dy/dx and evaluate it at x = 2.
3	(c) Let y = e^(3x)/x. Find dy/dx and evaluate it at x = 1.
4	(d) Let y = x^2 ln x. Find dy/dx and evaluate it at x = e.

2. Applications of differentiation: stationary points, tangents and related rates (16 marks)

Show f'(x) and justify each conclusion.

#	Question
5	(a) Let f(x) = x^3 - 6x^2 + 9x + 2. Find f'(x) and the coordinates of all stationary points.
6	(b) Find the equation of the tangent to y = x^2 + 3x at x = 2.
7	(c) Let g(x) = x^3 - 3x. Use the second derivative test to classify the stationary points at x = 1 and x = -1.
8	(d) The radius of a sphere increases at 0.5 cm per second. Find the rate of increase of its volume when the radius is 3 cm.

3. Integration and area between curves (16 marks)

Give definite integral answers to 4 decimal places and show the integration steps.

#	Question
9	(a) Find the indefinite integral of 6x^2 + 4x - 3 with respect to x.
10	(b) Evaluate the definite integral of 3x^2 from x = 1 to x = 2.
11	(c) Evaluate the integral of x e^(x^2) from x = 0 to x = 1.
12	(d) Find the area enclosed by y = x^2 and y = x for 0 <= x <= 1.

4. Binomial and Poisson distributions (18 marks)

Give probabilities to 4 decimal places and state the formula used.

#	Question
13	(a) X ~ B(10, 0.3). Find P(X = 3).
14	(b) Using (a), find the mean, variance and standard deviation of X.
15	(c) Y ~ Po(4). Find P(Y = 0) and P(Y >= 1).
16	(d) Cars pass a point on a road at an average of 2.5 per 5 minutes. Using a Poisson model, find the probability that exactly 2 cars pass in 5 minutes.

5. The normal distribution (16 marks)

Standard normal tables may be used: Phi(1) = 0.8413, Phi(1.5) = 0.9332, Phi(-1) = 0.1587, z(0.9) = 1.2816.

#	Question
17	(a) X ~ N(60, 8^2). Find P(X < 68).
18	(b) Using (a), find P(52 < X < 72).
19	(c) Using (a), find k such that P(X < k) = 0.9.
20	(d) Using (a), a sample of 16 is taken. Find P(sample mean > 63).

6. Point estimation, confidence interval and hypothesis testing (16 marks)

Give answers to 4 decimal places and state the test conclusion.

#	Question
21	(a) A sample is 12, 15, 9, 14, 10. Find the sample mean.
22	(b) Using (a), find the unbiased sample variance.
23	(c) A sample of 25 has mean 12 and standard deviation 5. Find the 95% confidence interval for the population mean.
24	(d) Test H0: mu = 50 against H1: mu != 50 with a sample of 36, mean 52.4 and standard deviation 9. Find the test statistic and conclude at the 5% level.