S5 Mathematics (Compulsory Part) Statistics and Probability Practice Paper

S5 | Mathematics (Compulsory Part) | Time: 75 minutes | Total: 100 marks | 24 questions

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Part A Averages and Dispersion (16 marks)

The weekly self-study hours of 8 students are: 12, 15, 9, 18, 14, 20, 11 and 13.

1. Find the mean of the data set. (4 marks)

Answer: Sum = 112; mean = 112 / 8 = 14 hours.

2. Find the median of the data set. (4 marks)

Answer: Sorted: 9, 11, 12, 13, 14, 15, 18, 20; median = (13 + 14) / 2 = 13.5 hours.

3. Find the standard deviation, correct to 2 decimal places. (4 marks)

Answer: Sum of squared deviations = 92; variance = 92 / 8 = 11.5; s.d. = sqrt(11.5) = 3.39 hours.

4. Find the range and the inter-quartile range. (4 marks)

Answer: Range = 20 - 9 = 11; lower half 9, 11, 12, 13 gives Q1 = 11.5; upper half 14, 15, 18, 20 gives Q3 = 16.5; IQR = 16.5 - 11.5 = 5 hours.

Part B Standard Scores and the Normal Distribution (16 marks)

The marks of a test follow approximately a normal distribution with mean 68 and standard deviation 12.

5. A student scores 80. Find the standard score. (4 marks)

Answer: z = (80 - 68) / 12 = 1, i.e. 1 standard deviation above the mean.

6. A student scores 53. Find the standard score. (4 marks)

Answer: z = (53 - 68) / 12 = -1.25, i.e. 1.25 standard deviations below the mean.

7. Using the 68-95-99.7 rule, estimate the proportion of students scoring between 56 and 80. (4 marks)

Answer: 56 = 68 - 12 (z = -1) and 80 = 68 + 12 (z = +1); about 68% of students lie between them.

8. There are 400 students. Estimate how many score above 92. (4 marks)

Answer: 92 = 68 + 2 x 12 (z = 2); about 2.5% lie above 2 standard deviations, so 400 x 0.025 = 10 students.

Part C Probability and Conditional Probability (18 marks)

A bag contains 5 red, 3 blue and 2 green balls. There are 10 balls in total, identical except in colour.

9. One ball is drawn at random. Find the probability that it is red. (4 marks)

Answer: P(red) = 5 / 10 = 0.5.

10. Two balls are drawn without replacement. Find the probability that both are red. (5 marks)

Answer: P(red, red) = (5/10) x (4/9) = 2/9 ≈ 0.22.

11. Given that the first ball drawn is red, find the probability that the second is also red. (4 marks)

Answer: 4 red balls remain among 9 balls, so P(2nd red | 1st red) = 4/9 = 0.44.

12. Two balls are drawn with replacement. Find the probability of getting at least one green ball. (5 marks)

Answer: P(no green) = (8/10)^2 = 0.64; P(at least one green) = 1 - 0.64 = 0.36.

Part D Permutations and Combinations (16 marks)

Among 7 students, 3 are boys and 4 are girls.

13. In how many ways can 5 of the 7 students be arranged in a row? (4 marks)

Answer: 7P5 = 7 x 6 x 5 x 4 x 3 = 2520 ways.

14. In how many ways can a group of 3 be chosen from the 7 students? (4 marks)

Answer: C(7,3) = 7! / (3! x 4!) = 35 ways.

15. In how many ways can all 7 students be arranged in a row if the 3 boys must stand together? (4 marks)

Answer: Treat the boys as one block: 3! x 5! = 6 x 120 = 720 ways.

16. In how many ways can a group of 3 be chosen so that it contains at least one girl? (4 marks)

Answer: C(7,3) - C(3,3) = 35 - 1 = 34 ways.

Part E Binomial Distribution (17 marks)

In a shooting game the hit rate is 0.3 and shots are independent. The player shoots 10 times. Let X be the number of hits.

17. Find P(X = 3). (5 marks)

Answer: P(X = 3) = C(10,3)(0.3)^3(0.7)^7 = 120 x 0.027 x 0.0824 = 0.27.

18. Find the expected value E(X) and the variance Var(X). (4 marks)

Answer: E(X) = np = 10 x 0.3 = 3; Var(X) = np(1 - p) = 10 x 0.3 x 0.7 = 2.1.

19. Find the standard deviation of X, correct to 2 decimal places. (4 marks)

Answer: sigma = sqrt(Var(X)) = sqrt(2.1) = 1.45.

20. Find P(X >= 1). (4 marks)

Answer: P(X >= 1) = 1 - P(X = 0) = 1 - (0.7)^10 = 1 - 0.03 = 0.97.

Part F Linear Regression and Correlation (17 marks)

The weekly study hours x and test scores y of 5 students are: (2, 52), (4, 60), (6, 68), (8, 74), (10, 80).

21. Find the mean of x and the mean of y. (4 marks)

Answer: x-bar = 30 / 5 = 6; y-bar = 334 / 5 = 66.8.

22. Find Sxy, Sxx and Syy. (4 marks)

Answer: Sxy = sum (x - x-bar)(y - y-bar) = 140; Sxx = 40; Syy = 492.8.

23. Find the correlation coefficient r (3 decimal places) and describe the strength. (5 marks)

Answer: r = Sxy / sqrt(Sxx x Syy) = 140 / sqrt(40 x 492.8) = 1, a very strong positive correlation.

24. Find the regression line y = a + bx and predict the score for 12 study hours. (4 marks)

Answer: b = 140 / 40 = 3.5; a = 66.8 - 3.5 x 6 = 45.8; y = 45.8 + 3.5 x; when x = 12, y = 87.8 marks.
