S6 Mathematics Extended Part Module 1 Practice Paper

S6 | Mathematics Extended Part Module 1 | Time: 75 minutes | Total: 100 marks | 24 questions

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Part 1: Binomial Theorem (16 marks)

State the formula used; give coefficients in simplest form.

1. In the expansion of (1 + 2x)^5, (a) find the coefficient of x^3; (b) find the coefficient of x^4.

Answer: (a) C(5,3) x 2^3 = 10 x 8 = 80. (b) C(5,4) x 2^4 = 5 x 16 = 80.

2. In the expansion of (2x + 3/x)^6, (a) find the constant term; (b) find the coefficient of x^2.

Answer: General term = C(6,r)(2x)^(6-r)(3/x)^r. (a) r = 3 gives the constant term = 4320. (b) r = 2 gives the coefficient of x^2 = 2160.

3. In the expansion of (1 + x)^n the coefficient of x^2 is 45. (a) Find n. (b) Find the coefficient of x^3.

Answer: (a) n(n-1)/2 = 45 so n = 10. (b) C(10,3) = 120.

4. In the expansion of (1 - x/2)^8, (a) find the coefficient of x^3; (b) find the sum of all the coefficients.

Answer: (a) C(8,3) x (-0.5)^3 = -7. (b) Put x = 1: sum = (0.5)^8 = 0.00390625.

Part 2: Exponential and Logarithmic Functions (16 marks)

Give answers to 4 decimal places unless exact.

5. Solve 2^x = 5, giving the answer to 4 decimal places.

Answer: x = ln5 / ln2 = 2.3219.

6. (a) Find log2 48 - log2 3. (b) Find log_a 9 / log_a 3 where a > 0, a not equal 1.

Answer: (a) log2(48/3) = log2 16 = 4. (b) log_a 9 = 2 log_a 3, so the ratio = 2.

7. A machine is worth V = 20000 e^(-0.08t) dollars after t years. (a) Find its value after 10 years. (b) Find the time needed for the value to halve.

Answer: (a) V = 20000 x e^(-0.8) = $8986.58. (b) t = ln2/0.08 = 8.6643 years.

8. Solve log x + log(x - 3) = 1 (common logarithm) and explain why one root is rejected.

Answer: x(x - 3) = 10 gives (x - 5)(x + 2) = 0. x = -2 is rejected because log x is undefined, so x = 5.

Part 3: Differentiation and Applications (18 marks)

Use the derivative to justify the nature of each stationary point.

9. Let y = x^3 - 6x^2 + 9x + 2. (a) Find the stationary points. (b) Determine their nature.

Answer: (a) dy/dx = 3x^2 - 12x + 9 = 0 gives x = 1, 3 with y = 6, 2. (b) y'' = 6x - 12 = -6 at x = 1 (maximum) and 6 at x = 3 (minimum).

10. Let y = x e^(2x). (a) Find dy/dx. (b) Find the slope of the curve at x = 0.

Answer: (a) dy/dx = e^(2x)(1 + 2x). (b) At x = 0 the slope = 1.

11. Let y = ln(x^2 + 1). (a) Find dy/dx. (b) Find the equation of the tangent at x = 1.

Answer: (a) dy/dx = 2x/(x^2 + 1). (b) At x = 1, y = ln 2 = 0.6931 and the slope is 1, so the tangent is y = x - 0.3069.

12. The point P(3, 9) lies on the curve y = x^2. (a) Find the tangent at P. (b) Find the normal at P.

Answer: (a) dy/dx = 2x = 6 at P, so the tangent is y = 6x - 9. (b) The normal has slope -1/6, giving x + 6y = 57.

Part 4: Integration and Area (18 marks)

Add a constant of integration for indefinite integrals.

13. Evaluate the integral of (3x^2 - 2x) from x = 1 to x = 4.

Answer: The antiderivative is x^3 - x^2, giving (64 - 16) - (1 - 1) = 48.

14. (a) Find the integral of x e^(x^2) dx. (b) Hence evaluate it from 0 to 1 to 4 decimal places.

Answer: (a) With u = x^2 the integral is (1/2)e^(x^2) + C. (b) The value is (e - 1)/2 = 0.8591.

15. Find the area enclosed by the curve y = x^2 and the line y = 2x.

Answer: The curves meet at x = 0 and 2. Area = integral from 0 to 2 of (2x - x^2) = 4 - 8/3 = 1.3333.

16. Given the integral of 2x from 0 to k equals 16 with k > 0. (a) Find k. (b) Hence find the integral of (2x + 3) from 0 to k.

Answer: (a) k^2 = 16 so k = 4. (b) 16 + 3 x 4 = 28.

Part 5: Probability and Discrete Random Variables (16 marks)

Give probabilities to 4 decimal places unless a fraction is exact.

17. A discrete random variable X has P(X = 0) = 0.3, P(X = 1) = 0.5, P(X = 2) = 0.2. (a) Find E(X). (b) Find Var(X).

Answer: (a) E(X) = 0.90. (b) E(X^2) = 1.3 so Var(X) = 1.3 - 0.81 = 0.49.

18. A bag has 5 red and 3 blue balls; 2 are drawn without replacement. (a) Find the probability that both are red. (b) Find the probability of one red and one blue.

Answer: (a) C(5,2)/C(8,2) = 10/28 = 0.3571. (b) 15/28 = 0.5357.

19. Given P(A) = 0.6, P(B) = 0.5 and A, B are independent. (a) Find P(A and B). (b) Find P(A or B). (c) Find P(A | B).

Answer: (a) 0.3. (b) 0.8. (c) By independence P(A | B) = P(A) = 0.6.

20. A game pays $10 on a win (probability 0.2) and costs $3 on a loss (probability 0.8). (a) Find the expected gain per game. (b) What prize would make the game fair?

Answer: (a) 10 x 0.2 - 3 x 0.8 = -0.4, an expected loss of $0.4 per game. (b) For a fair game 0.2x - 2.4 = 0, so x = $12.

Part 6: Binomial, Poisson and Normal Distributions (16 marks)

The standard normal table may be used.

21. Let X ~ B(10, 0.3). (a) Find P(X = 3). (b) Find the mean and variance of X.

Answer: (a) C(10,3) x 0.3^3 x 0.7^7 = 0.2668. (b) Mean = 3, variance = 2.1.

22. A hotline receives 4 calls per minute on average, Y ~ Po(4). (a) Find the probability of no call in a minute. (b) Find the probability of at least one call.

Answer: (a) P(Y = 0) = e^(-4) = 0.0183. (b) P(Y >= 1) = 1 - 0.0183 = 0.9817.

23. Marks are normally distributed with mean 60 and standard deviation 8. (a) Find the proportion scoring above 72. (b) Find the mark exceeded by only 10% of candidates.

Answer: (a) z = 1.5 so the proportion = 0.0668. (b) z = 1.2816 gives the mark 60 + 8 x 1.2816 = 70.25.

24. A fair coin is tossed 100 times. Use a normal approximation with continuity correction to estimate the probability of 45 to 55 heads.

Answer: Mean 50, standard deviation 5; with continuity correction 44.5 to 55.5, z = +/-1.1, so the probability = 0.7287.
