S6 Mathematics Extended Part Module 1 Practice Paper

S6 | Mathematics Extended Part Module 1 | Time: 75 minutes | Total: 100 marks | 24 questions

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Part 1: Binomial Theorem (16 marks)

State the formula used; give coefficients in simplest form.

1. In the expansion of (1 + 2x)^5, (a) find the coefficient of x^3; (b) find the coefficient of x^4.

2. In the expansion of (2x + 3/x)^6, (a) find the constant term; (b) find the coefficient of x^2.

3. In the expansion of (1 + x)^n the coefficient of x^2 is 45. (a) Find n. (b) Find the coefficient of x^3.

4. In the expansion of (1 - x/2)^8, (a) find the coefficient of x^3; (b) find the sum of all the coefficients.

Part 2: Exponential and Logarithmic Functions (16 marks)

Give answers to 4 decimal places unless exact.

5. Solve 2^x = 5, giving the answer to 4 decimal places.

6. (a) Find log2 48 - log2 3. (b) Find log_a 9 / log_a 3 where a > 0, a not equal 1.

7. A machine is worth V = 20000 e^(-0.08t) dollars after t years. (a) Find its value after 10 years. (b) Find the time needed for the value to halve.

8. Solve log x + log(x - 3) = 1 (common logarithm) and explain why one root is rejected.

Part 3: Differentiation and Applications (18 marks)

Use the derivative to justify the nature of each stationary point.

9. Let y = x^3 - 6x^2 + 9x + 2. (a) Find the stationary points. (b) Determine their nature.

10. Let y = x e^(2x). (a) Find dy/dx. (b) Find the slope of the curve at x = 0.

11. Let y = ln(x^2 + 1). (a) Find dy/dx. (b) Find the equation of the tangent at x = 1.

12. The point P(3, 9) lies on the curve y = x^2. (a) Find the tangent at P. (b) Find the normal at P.

Part 4: Integration and Area (18 marks)

Add a constant of integration for indefinite integrals.

13. Evaluate the integral of (3x^2 - 2x) from x = 1 to x = 4.

14. (a) Find the integral of x e^(x^2) dx. (b) Hence evaluate it from 0 to 1 to 4 decimal places.

15. Find the area enclosed by the curve y = x^2 and the line y = 2x.

16. Given the integral of 2x from 0 to k equals 16 with k > 0. (a) Find k. (b) Hence find the integral of (2x + 3) from 0 to k.

Part 5: Probability and Discrete Random Variables (16 marks)

Give probabilities to 4 decimal places unless a fraction is exact.

17. A discrete random variable X has P(X = 0) = 0.3, P(X = 1) = 0.5, P(X = 2) = 0.2. (a) Find E(X). (b) Find Var(X).

18. A bag has 5 red and 3 blue balls; 2 are drawn without replacement. (a) Find the probability that both are red. (b) Find the probability of one red and one blue.

19. Given P(A) = 0.6, P(B) = 0.5 and A, B are independent. (a) Find P(A and B). (b) Find P(A or B). (c) Find P(A | B).

20. A game pays $10 on a win (probability 0.2) and costs $3 on a loss (probability 0.8). (a) Find the expected gain per game. (b) What prize would make the game fair?

Part 6: Binomial, Poisson and Normal Distributions (16 marks)

The standard normal table may be used.

21. Let X ~ B(10, 0.3). (a) Find P(X = 3). (b) Find the mean and variance of X.

22. A hotline receives 4 calls per minute on average, Y ~ Po(4). (a) Find the probability of no call in a minute. (b) Find the probability of at least one call.

23. Marks are normally distributed with mean 60 and standard deviation 8. (a) Find the proportion scoring above 72. (b) Find the mark exceeded by only 10% of candidates.

24. A fair coin is tossed 100 times. Use a normal approximation with continuity correction to estimate the probability of 45 to 55 heads.
