S6 M2 Practice Paper 1 | Induction, binomial, trigonometry, calculus and vectors

Senior Secondary Mathematics (Extended Part Module 2)

S6 M2 practice paper with worked answers: induction and series sums, binomial theorem, trig identities, limits, differentiation, definite integrals and vectors.

1. Mathematical induction and sequences (16 marks)

Show working; give answers as exact integers.

#	Question	Answer and marking notes
1	(a) Find the sum 1 + 3 + 5 + … + 39 and state the number of terms.	Number of terms n = (39 + 1) / 2 = 20; sum = n^2 = 20^2 = 400.
2	(b) A geometric sequence has first term 3 and common ratio 2. Find T8 and S8.	T8 = 3 x 2^7 = 384; S8 = 3(2^8 - 1) / (2 - 1) = 765.
3	(c) An arithmetic sequence has first term 7 and common difference 5. Find T20 and S20.	T20 = 7 + 19 x 5 = 102; S20 = 20(7 + 102) / 2 = 1090.
4	(d) Prove by induction that 1^3 + 2^3 + … + n^3 = [n(n+1)/2]^2, then check with n = 10.	Base n = 1: LHS = 1, RHS = 1^2 = 1; assuming true for n = k, adding (k+1)^3 gives [(k+1)(k+2)/2]^2, so the statement holds. For n = 10: LHS = 3025, RHS = (55)^2 = 3025.

2. Binomial theorem (18 marks)

Write the general term; coefficients are integers.

#	Question	Answer and marking notes
5	(a) Find the coefficient of x^3 in the expansion of (1 + 2x)^8.	General term = C(8,r)(2x)^r; for r = 3 the coefficient is C(8,3) x 2^3 = 56 x 8 = 448.
6	(b) Find the constant term in the expansion of (x - 3/x)^6.	General term = C(6,r)(-3)^r x^(6-2r); setting 6 - 2r = 0 gives r = 3, so the constant term is C(6,3) x (-3)^3 = 20 x (-27) = -540.
7	(c) Find the coefficient of x^7 in the expansion of (2 + x)^10.	Coefficient = C(10,7) x 2^3 = 120 x 8 = 960.
8	(d) Find the sum of all coefficients in (2 + x)^10 and state the method used.	Put x = 1; the sum of all coefficients is (2 + 1)^10 = 3^10 = 59049.

3. Trigonometric identities and equations (16 marks)

θ is acute; give angles in degrees.

#	Question	Answer and marking notes
9	(a) Given sin θ = 3/5 and θ acute, find cos θ and tan θ.	cos θ = 4/5 = 0.8; tan θ = 3/4 = 0.75 (a 3-4-5 triangle).

#	Question	Answer and marking notes
10	(b) Using (a), find the values of sin 2θ and cos 2θ.	sin 2θ = 2 sinθ cosθ = 2 x 0.6 x 0.8 = 0.96; cos 2θ = 1 - 2 sin^2θ = 1 - 0.72 = 0.28.
11	(c) Solve 2 sin^2 x - sin x - 1 = 0 for 0° ≤ x ≤ 360°.	(2 sinx + 1)(sinx - 1) = 0, so sinx = -1/2 or 1; hence x = 210°, 330°, 90°.
12	(d) Given tan A = 1/2 and tan B = 1/3, find tan(A + B) and the corresponding acute angle.	tan(A+B) = (1/2 + 1/3) / (1 - 1/6) = (5/6) / (5/6) = 1, so A + B = 45°.

4. Limits and differentiation (18 marks)

Show working; give values to 3 decimal places.

#	Question	Answer and marking notes
13	(a) Evaluate lim (x→3) (x^2 - 9)/(x - 3).	Factorise the numerator as (x - 3)(x + 3), cancel to get x + 3, then substitute x = 3: 6.
14	(b) Let y = x^3 e^(2x). Find dy/dx and the slope at x = 1.	dy/dx = 3x^2 e^(2x) + 2x^3 e^(2x) = x^2 e^(2x)(3 + 2x); at x = 1 the slope is 5e^2 = 36.945.
15	(c) Let y = ln(x^2 + 1). Find dy/dx and the slope at x = 2.	dy/dx = 2x/(x^2 + 1); at x = 2 the slope is 4/5 = 0.8.
16	(d) For the curve y = x^3 - 3x^2 + 2, find all stationary points and classify them.	y’ = 3x^2 - 6x = 3x(x - 2), so x = 0 and 2; y” = 6x - 6, at x = 0 it is -6 (maximum, y = 2), at x = 2 it is +6 (minimum, y = -2).

5. Definite integrals and areas (16 marks)

Give the antiderivative; areas to 3 decimal places.

#	Question	Answer and marking notes
17	(a) Evaluate the integral of (3x^2 + 2x) from 0 to 2.	Antiderivative x^3 + x^2; evaluating gives (8 + 4) - 0 = 12.
18	(b) Evaluate the integral of 1/x from 1 to e.	Antiderivative ln x; the value is ln e - ln 1 = 1.
19	(c) Evaluate the integral of x e^x from 0 to 1.	By parts, the antiderivative is e^x(x - 1); substituting gives e(0) - 1(-1) = 1.
20	(d) Find the area enclosed by y = 2x and y = x^2.	The curves meet at x = 0 and 2; area = integral of (2x - x^2) from 0 to 2 = 4 - 8/3 = 1.333 square units.

6. Matrices, determinants and vectors (16 marks)

Give vectors in components; angles to 1 decimal place.

#	Question	Answer and marking notes
21	(a) Let A = [[2, 1], [3, 4]]. Find det A and the inverse of A.	det A = 2 x 4 - 1 x 3 = 5; the inverse of A is (1/5)[[4, -1], [-3, 2]].
22	(b) Let B = [[1, 2], [2, 1]]. Find det B and state whether B is invertible.	det B = 1 x 1 - 2 x 2 = -3, which is non-zero, so B is invertible.
23	(c) Let a = (2, -1, 3) and b = (1, 4, -2). Find a.b and the angle between them.	a.b = 2 - 4 - 6 = -8; |a| = root 14, |b| = root 21; cosθ = -8/root 294 = -0.467, so θ is about 117.8°.
24	(d) Using a and b in (c), find a x b.	a x b = ((-1)(-2) - (3)(4), (3)(1) - (2)(-2), (2)(4) - (-1)(1)) = (-10, 7, 9).