S6 Mathematics Module 2 Practice Paper 1 | limits, differentiation, integration, matrices, vectors, binomial, optimisation

Senior Secondary Mathematics

S6 Mathematics Module 2 practice paper with worked solutions: limits, differentiation, integration, matrices, vectors, binomial distribution, optimisation.

1. Limits and differentiation techniques (18 marks)

Let f(x) = e^(2x) ln x, let C be the curve x^2 + y^2 = 25, and let g(x) = x^3 - 3x.

#	Question
1	(a) Find lim(x->0) (sin 3x) / x.
2	(b) Find f'(x) and the exact value of f'(1).
3	(c) At the point (3, 4) on C, find dy/dx.
4	(d) Find the maximum and minimum points of g(x) and determine their nature.

2. Integration techniques and volume of revolution (16 marks)

Evaluate the following, giving answers correct to three decimal places where appropriate.

#	Question
5	(a) Find the exact value of the integral of (3x^2 + 2x) dx from 1 to 2.
6	(b) Use integration by parts to find the exact value of the integral of x e^x dx from 0 to 1.
7	(c) Use substitution to find the integral of 2x/(x^2 + 1) dx from 0 to 1, correct to three decimal places.
8	(d) The curve y = sqrt(x) from x = 0 to x = 4 is rotated about the x-axis. Find the volume of the solid in terms of pi.

3. Matrices and systems of linear equations (18 marks)

Let A be the 2 by 2 matrix with rows (2, 1) and (5, 3). Consider the system 2x + y = 7 and 5x + 3y = 18.

#	Question
9	(a) Find det A and the inverse of A.
10	(b) Use the inverse matrix to solve the system.
11	(c) Evaluate the determinant of the 3 by 3 matrix with rows (1, 2, 3), (0, 1, 4) and (5, 6, 0).
12	(d) If the determinant of a 3 by 3 matrix is 0, what can be concluded about the corresponding system of equations?

4. Dot products, cross products and areas (18 marks)

Let vector a be (3, -1, 2) and vector b be (1, 4, -2).

#	Question
13	(a) Find the dot product of a and b.
14	(b) Find the angle between a and b, correct to one decimal place.
15	(c) Find the cross product of a and b.
16	(d) Find the area of the parallelogram with a and b as adjacent sides, correct to two decimal places.

5. The binomial distribution (16 marks)

The defective rate of components from a factory is 5 per cent. Twenty components are chosen at random and X, the number of defectives, follows a binomial distribution.

#	Question
17	(a) Find the probability that X equals 0, correct to four decimal places.
18	(b) Find the probability that X is at least 1, correct to four decimal places.
19	(c) Find the mean, variance and standard deviation of X, giving the standard deviation to four decimal places.
20	(d) Find the probability that X equals 2, correct to four decimal places.

6. Optimising the surface area of an open tank (16 marks)

An open-top rectangular tank has a square base of side x metres and a fixed capacity of 32 cubic metres. Its height is h metres.

#	Question
21	(a) Express h in terms of x and write down the surface area A.
22	(b) Find the value of x that minimises A and the corresponding h.
23	(c) Verify that this value of x gives a minimum and find the minimum surface area.
24	(d) If the material costs 150 dollars per square metre, find the material cost at the minimum surface area, and the balance left from a budget of 8000 dollars.