S4 Mathematics Extended Module 2 Practice Paper: Mathematical Induction, Binomial Theorem, Compound Angles and Trigonometric Equations, Vectors, Matrices and Linear Transformations, Limits, Derivatives and Integration

S4 | Mathematics Extended Part (Module 2) | Time: 75 minutes | Total: 100 marks | 24 questions

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Part 1: Mathematical Induction (15 marks)

State the base case, the induction hypothesis and the induction step clearly.

1. Prove by mathematical induction that 1 + 3 + 5 + ... + (2n - 1) = n^2 for all positive integers n. [3]

2. Prove that 2^n > n^2 for all integers n >= 5. [4]

3. Prove that the sum of r x 2^r from r = 1 to n equals (n - 1)2^(n+1) + 2 for all positive integers n. [4]

4. Prove that 7^n - 1 is divisible by 6 for all positive integers n. [4]

Part 2: Binomial Theorem (17 marks)

Simplify every term; leave coefficients as integers or fractions in lowest terms.

5. Expand (2x - 3)^4. [5]

6. Find the coefficient of x^6 in the expansion of (x^2 + 2/x)^6. [4]

7. Find the constant term in the expansion of (2x - 1/(2x))^8. [4]

8. Given that the coefficients of x and x^2 in the expansion of (1 + kx)^n are 24 and 252 respectively, find n and k. [4]

Part 3: Compound Angles and Trigonometric Equations (17 marks)

Unless stated otherwise, give angles in degrees correct to 3 significant figures.

9. Given sin A = 3/5 and cos B = 5/13, where A and B are acute, find the exact value of sin(A + B). [4]

10. Solve 2sin^2 θ + sin θ - 1 = 0 for 0 <= θ < 360 degrees. [4]

11. Simplify sin 2θ / (1 + cos 2θ) and state the restriction on θ. [4]

12. In triangle ABC, a = 7, b = 9 and C = 60 degrees. Find c and the area of the triangle, correct to 3 significant figures. [5]

Part 4: Vectors (17 marks)

Write vectors as ordered pairs; give magnitudes as exact values or 3 significant figures.

13. Given a = (3, -1) and b = (-2, 4), find 2a - b, |2a - b| and the unit vector in the direction of 2a - b. [4]

14. Given that a = (1, 2) and b = (k, 3) are perpendicular, find k. [4]

15. Find the angle between the vectors (1, 1) and (1, 0). [4]

16. Given A(1, 2), B(4, 6) and C(7, 2), find the area of triangle ABC. [5]

Part 5: Matrices and Linear Transformations (17 marks)

Show the multiplication step by step; do not skip working by using a calculator matrix function.

17. Let A = [[2, 1], [3, 4]] and B = [[1, 0], [2, 1]]. Find AB and BA, and hence explain why matrix multiplication is not commutative. [4]

18. Find the inverse of the matrix [[3, 1], [2, 4]]. [4]

19. Solve the system 2x + y = 7 and x - y = 2 by the matrix method. [4]

20. The matrix M = [[0, -1], [1, 0]] represents an anticlockwise rotation of 90 degrees. Find the image of (3, 2) under M and state the transformation represented by M^2. [5]

Part 6: Limits, Derivatives and Integration (17 marks)

Show the simplification for limits; add the constant C for indefinite integrals.

21. Evaluate lim (x -> 3) (x^2 - 9)/(x - 3). [4]

22. Find d/dx of x^2 ln x. [4]

23. Evaluate the integral from 1 to 2 of (3x^2 + 2/x) dx, giving the exact value. [4]

24. Find the equation of the tangent to the curve y = x^3 - 2x at x = 1. [5]
