S6 Mathematics Extended Part Module 2 Practice Paper: Binomial Theorem and Mathematical Induction, Complex Numbers and Trigonometry, Limits and Differentiation, Applications of Differentiation, Integration and its Applications, Matrices and Vectors

S6 | Mathematics (Module 2) | Time: 75 minutes | Total: 100 marks | 24 questions

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Part A: Binomial Theorem and Mathematical Induction (16 marks)

You need not expand every term; just write the answer.

1. Expand (1+2x)^6 and find the coefficient of x^3. [4]

Answer: Answer: 160 (C(6,3) x 2^3 = 20 x 8).

2. Find the coefficient of x^2 in the expansion of (x-3)^5. [4]

Answer: Answer: -270 (take r = 3: C(5,3) x (-3)^3).

3. Prove by induction that 1+3+5+...+(2n-1)=n^2; which odd number must be added when going from n=k to n=k+1? [4]

Answer: Answer: 2k+1, the (k+1)th odd number.

4. Find the value of the sum of C(6,r) for r = 0 to 6. [4]

Answer: Answer: 64 (= 2^6).

Part B: Complex Numbers and Trigonometry (16 marks)

Give exact values in simplest form.

5. Simplify (3+2i)(1-4i) and give the answer in the form a+bi. [4]

Answer: Answer: 11-10i.

6. Find |3+4i|. [4]

Answer: Answer: 5.

7. If sinθ=3/5 and θ lies in the second quadrant, find the exact value of cosθ. [4]

Answer: Answer: -4/5 (cosine is negative in the second quadrant).

8. Given sinA=3/5 and cosB=5/13 with A and B acute, find the exact value of cos(A+B). [4]

Answer: Answer: -16/65 (cosA=4/5, sinB=12/13).

Part C: Limits and Differentiation (16 marks)

Evaluate the limit or derivative and show the main steps.

9. Find lim (x->2) (x^2-4)/(x-2). [4]

Answer: Answer: 4 (cancel x-2 to get x+2).

10. Find lim (x->0) sin3x/x. [4]

Answer: Answer: 3 (= 3 x lim sin3x/3x).

11. Find d/dx (x^3 ln x). [4]

Answer: Answer: 3x^2 ln x + x^2 (product rule).

12. Find d/dx (2x+1)^5. [4]

Answer: Answer: 10(2x+1)^4 (chain rule).

Part D: Applications of Differentiation (16 marks)

Give coordinates or exact values in your answers.

13. Find the slope of the tangent to the curve y=x^3-3x at x=2. [4]

Answer: Answer: 9 (dy/dx = 3x^2 - 3).

14. Find the x-coordinates of all stationary points of the curve y=x^4-8x^2. [4]

Answer: Answer: x=-2, 0, 2 (from 4x^3-16x=0).

15. Find the maximum and minimum values of f(x)=x^3-6x^2+9x. [4]

Answer: Answer: maximum 4 at x=1, minimum 0 at x=3.

16. Find the x-coordinate of the point of inflexion of y=x^3-3x^2+2x. [4]

Answer: Answer: x=1 (y" = 6x - 6 = 0).

Part E: Integration and its Applications (16 marks)

Add the constant C for indefinite integrals; give exact values for areas and volumes.

17. Evaluate the integral of 3x^2 from 0 to 1. [4]

Answer: Answer: 1 ([x^3] from 0 to 1).

18. Find the integral of x cos x dx. [4]

Answer: Answer: x sin x + cos x + C (integration by parts).

19. Find the area of the region bounded by y=x^2 and y=x for 0<=x<=1. [4]

Answer: Answer: 1/6 (integrate x - x^2).

20. The region under y=x for 0<=x<=2 is rotated about the x-axis; find the volume. [4]

Answer: Answer: 8π/3 (pi times the integral of x^2).

Part F: Matrices and Vectors (20 marks)

Give complete answers; write vectors in component form.

21. Solve the simultaneous equations 2x+y=7 and x-y=2. [5]

Answer: Answer: x=3, y=1.

22. Find the inverse of the matrix A = [[2,1],[5,3]]. [5]

Answer: Answer: [[3,-1],[-5,2]] (determinant = 1).

23. Given a=(1,2,2) and b=(2,-1,2), find a.b and |a|. [5]

Answer: Answer: a.b = 4, |a| = 3.

24. Given a=(1,0,0) and b=(0,3,4), find a x b and |a x b|. [5]

Answer: Answer: (0,-4,3), |a x b| = 5 (the vectors are perpendicular).
